Sample Logic

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Abstract. The need for a 'many-valued logic' in linguistics has been evident since the 1970s, but there was lack of clarity as to whether it should come from the family of fuzzy logics or from the family of probabilistic logics. In this regard, Fine (1975) and Kamp (1975) pointed out undesirable effects of fuzzy logic (the failure of idempotency and coherence) which kept two generations of linguists and philosophers at arm's length. (Another unwanted feature of fuzzy logic is the property of truth-functionality.) While probabilistic logic is not fraught by the same problems, its lack of constructiveness, that is, its inability to compose complex truth degrees from atomic truth degrees, did not make it more attractive to linguists either. In the absence of a clear perspective in 'many-valued logic', scholars chose to proliferate ontologies grafted atop the classical bivalent logic: ontologies for truth, individuals, events, situations, possible worlds and degrees. The result has been a collection of incompatible classical logics. In this paper, I present Sample Logic, in particular its semantics (not its axiomatization). Sample Logics is a member of the family of probabilistic logics, which is constructive without being truth-functional. More specifically, I integrate all the important linguistic data on which the classical logics are predicated. The concepts of (in)dependency and conditional (in)dependency are the cornerstones of Sample Logic.

Keywords: Sample Logic, fuzzy logic, probabilistic logic, independency, conditional independency

1. Introduction

In this introduction, I critically survey the relevance of Classical Bivalent Logic (1.1), Fuzzy Logic (1.2) and Probabilistic Logic (1.3) for linguistics, displaying logical properties pertinent for the purpose of language analysis in an overview table (1.4).

Disclaimer: Early vanguards (Reichenbach 1949) used the term 'many-valued logic' to denote logics with more than two truth values, encompassing both fuzzy logics and probabilistic logics. Somehow in subsequent usage, the term came to only refer to logics that are truth-functional, mainly to fuzzy logics (Gottwald 2017), although the term itself does not underwrite this limitation. In this paper, I continue to use 'many-valued logic' as an inclusive term of fuzzy and probabilistic logics.

1.1 The case against Classical Logic

The main charges against Classical Logic are its *evaluation gaps* and its *truth-functionality*.

With respect to the first charge, so-called borderline cases (Sorensen 2018) or borderline contradictions (Alxatib et al. 2013; Cobreros et al. 2012) represent difficult examples for Classical Logic, given that there are only two truth values to draw from. Knowing that the median height of all 497 registered NBA basketball players is 199.8 cm, Jimmy Butler of Miami Heat (with his 199.8 cm) represents a borderline case of tallness.

(1) a. Jimmy Butler of Miami Heat is tall.
    b. Jimmy Butler of Miami Heat is tall and not tall.

Super- and subvaluationists attempt to interpret the above propositions in Classical Logic. According to the supervaluationists, borderline statements and borderline contradictions lack a truth-value (such as sentences with a truncated reference 'The king of France is bald'), whereas the subvaluationists claim that examples in (1) can be both true and false. Ostensibly, both approaches advocate the existence of a third truth value under the guise of terms like 'gap' or 'glut' rather than to make a genuine case for bivalent logic.[1]

Truth-functionality denotes the principle that the truth of compound sentences is determined by the truth values of its component sentences and remains unaffected if one of the component

[1] In two additional approaches, epistemicism (Williamson 1994) and contextualism (Price 2008), the truth of (1a) and (1b) would depend on the speaker's knowledge or on a particular context. Since speaker knowledge and context are framed by the mention of "Miami Heat", these models bear the risk of assigning inappropriate truth values.

sentences is replaced by another sentence with the same value (Gottwald 2017). However, this property is undesirable in linguistic analysis because it is not only the truth values of the component sentences that matter, but also their mutual relationship. Consider the following examples.

(2) a. Nicolas Batum is taller than Cody Zeller and Joe Chealey is taller than Terry Rozier.
b. Joe Chealey is taller than Caleb Martin and Joe Chealey is taller than Terry Rozier.
c. Joe Chealey is taller than Caleb Martin and Terry Rozier.

All players belong to the NBA basketball team 'Charlotte Hornets'.[2] In accordance with Classical Logic, the first compound sentence is false if Nicolas Batum or Joe Chealey (or both) happen to be smaller than or as tall as their respective standard of comparison (which is the case). As *Nicolas Batum is taller than Cody Zeller* in (2a) and *Joe Chealey is taller than Caleb Martin* in (2b) are both false, replacing the first by the second provides (2b), which should have the same truth value as (2a), thanks to truth functionality. Yet, most people would interpret (2b) cumulatively as (2c) and judge (2c) differently from (2a) as neither true nor false, or as 'half-true', since it is evident that both component sentences in (2b)/(2c) share the same object of comparison, Joe Chealey, and that they are therefore correlated. In (2a), the two component sentences are uncorrelated or independent. Classical Logic does not draw a distinction between mutually dependent and independent sentences. Instead, both receive the same truth-functional treatment, which constitutes a major flaw.

1.2 The case against Fuzzy Logic

In his seminal paper *Fuzzy Sets* (1965), electric engineer Lotfi Zadeh pioneered the concept of Fuzzy Logic. Taking truth degrees from the real number interval [0, 1], he defined the truth of a conjunctive statement as the minimum of the two component truth degrees (1965: 341). Correspondingly, Fine (1975) and Kamp (1975) examined this version of Fuzzy Logic and rejected it as unsuitable for use in linguistics. Kamp argued (p. 131) that the way Zadeh designed Fuzzy Logic does not **simultaneously** satisfy the properties of conjunctive idempotency ($[\![\phi \wedge \phi]\!] = [\![\phi]\!]$) and of conjunctive coherence ($[\![\phi \wedge \neg\phi]\!] = 0$). Put differently, if we assume conjunctive idempotency, then conjunctive coherence will fail. On the other hand, if we assume conjunctive coherence, then conjunctive idempotency will fail. Yet we need both in linguistics and philosophy. Kamp's argument convinced two generations of philosophers and linguists to stay away from Fuzzy Logic.

Fuzzy Logic was later revolutionized by the works of Petr Hájek (1998), who based Fuzzy Logic (in fact, the family of fuzzy logics) on the notion of *t-norm* (p. 28). In particular, conjunctions in Fuzzy Logic are defined by the notion of *t-norm*: $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \otimes [\![\psi]\!]$.

(2) A **t-norm** is a continuous function $\otimes : [0,1]^2 \to [0,1]$ satisfying the following conditions:

a. $\otimes$ is commutative and associative:
- $x \otimes y = y \otimes x$
- $(x \otimes y) \otimes z = x \otimes (y \otimes z)$

b. $\otimes$ is non-decreasing in both arguments:
- $x_1 \leq x_2 \Rightarrow x_1 \otimes y \leq x_2 \otimes y$
- $y_1 \leq y_2 \Rightarrow x \otimes y_1 \leq x \otimes y_2$

c. $\otimes$ is absorbing:
- $1 \otimes x = x$
- $0 \otimes x = 0$

[2] The height of the players is as follows: Cody Zeller 213 cm; Nicolas Batum 206 cm; Caleb Martin 196 cm; Joe Chealey 193 cm; and Terry Rozier 185 cm.

In an interesting paper, Sauerland (2011) renewed Kamp's argument against Fuzzy Logic based on t-norms by using fuzzy negation and Brouwer's Fixed Point Theorem,[3] but there is a gap in his proof. Instead, I put forward a direct way of rebuilding Kamp's case against Fuzzy Logic, which makes full use of the notion of t-norms. To know more about the proof of the following lemma and details of Sauerland's argument, refer to appendix, item A.

(3) *Lemma*:

Let $\otimes:[0,1]^2 \to [0,1]$ be a t-norm on which a fuzzy evaluation $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \otimes [\![\psi]\!]$ is based. **If** the evaluation $[\![\cdot]\!]$ simultaneously satisfy the properties of *idempotency* ($[\![\phi \wedge \phi]\!] = [\![\phi]\!]$) and of *coherence* ($[\![\phi \wedge \neg\phi]\!] = 0$), **then** the negation is reduced to 0 and 1 (for all ϕ: $[\![\neg\phi]\!]$ = 0 or 1).

In addition to these problems,[4] Fuzzy Logic is truth-functional like Classical Logic, which further disqualifies it from being applied to linguistics (despite the fact that this feature appears to be a chief asset in engineer sciences).

1.3 The case for Probabilistic Logic

The Russian mathematician Andrei Kolmogorov first axiomatized Probability Theory in 1933. The set-theoretic formulation of his axioms assumes a population **Ω**, a system **F** of subsets of **Ω**, called algebra,[5] and a probability (measure) function **P**: **F** → [0,1], satisfying the axioms in (4). Adam (1998) and other logicians (Demey et al. 2019) elaborated upon a logical formulation of these axioms that omits Kolmogorov's set structure. With a view to achieving this objective, they linked their version of Probabilistic Logic expressively to Classical Bivalent Logic, which is therefore a (non-truth-functional) extension of it. A probabilistic evaluation $[\![\cdot]\!]$: SENT → [0,1] satisfies the following axioms of Kolmogorov.

(4)

	Logical Version (Adam 1998)	*Set-theoretic Version*
K1:	$0 \le [\![\phi]\!] \le 1$	0 ≤ **P**(A) ≤ 1 for A ∈ **F**
K2:	If ϕ is a tautology in Classical Logic, then $[\![\phi]\!] = 1$	**P**(**Ω**) = 1
K3:	If ϕ and ψ are logically incoherent in Classical Logic, then $[\![\phi \vee \psi]\!] = [\![\phi]\!] + [\![\psi]\!]$.	If A ∩ B = ∅, then **P**(A ∪ B) = **P**(A)+ **P**(B).

The lack of success attained by Probabilistic Logic in linguistics is primarily attributed to the lack of constructiveness. Probabilities or truth degrees are defined deductively with few instructions attached on how to construe compound truth degrees. Exceptions are the subclass of disjunctive sentences, namely those whose conjunctive counterpart is incoherent (K3), and the subclass of independent conjunctive sentences (where $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cdot [\![\psi]\!]$). The probabilistic conditional $[\![\phi \to \psi]\!]$ is partly defined in a constructive manner as the quotient of $[\![\phi \wedge \psi]\!]$ and $[\![\phi]\!]$, but we still cannot determine the conjunctive probability. In my

[3] Smith (2015: 1275-76) suggested using different fuzzy logics for different language data: to involve Łukasiewicz Fuzzy Logic where only coherence and continuous negation are needed, and to employ Gödel Fuzzy Logic where only coherence and idempotency are requested. I am somewhat critical of this approach as this would lead to the same kind of fragmentation I try to argue against in the case of Classical Logic (see abstract and conclusion).

[4] See for example Border (1989: 28).

[5] A system of subsets **F** is an algebra iff (i) **Ω** ∈ **F**; (ii) for A ∈ **F**: A^C ∈ **F**; (iii) for A, B ∈ **F**: A ∪ B ∈ **F**. A system of subsets **F** is a σ-algebra iff (i) and (ii) hold and not only the union of two but an infinite union belongs to **F**. In general Probability Theory, the axioms are based on σ-algebras. As we deal with finite language expressions, the notion of an algebra is sufficient. See Kolmogorov (1933) and von Plato (2005).

understanding, compound truth degrees are constructive if they are defined as a function of the component degrees and possibly of some other parameter (e.g. dependence) which may or may not be truth-functional. Constructiveness is thus entailed by truth-functionality but does not entail it.

The linkage between Probabilistic and Classical Logic is a problematic assumption.[6] The probabilistic conditional $[\![\phi \rightarrow \psi]\!]$, for instance, is defined as the quotient of $[\![\phi \wedge \psi]\!]$ and $[\![\phi]\!]$. It differs from the material conditional in Classical Logic, which is defined as a disjunction $[\![\neg\phi \vee \psi]\!]$. The truth degrees of both expressions diverge in general, which poses a problem for K2 (see also section 3.3).[7] In fact, there is a simple way of formulating the Kolmogorov axioms that is equivalent to the set-theoretic version and that avoids the link to Classical Logic. If we replace "**P**(**Ω**) = 1" by "**P**(A $\cup$ A^C) = 1" in the axiom K2, we can then define a probabilistic evaluation $[\![\cdot]\!]$: SENT $\rightarrow$ [0,1] as one that satisfies the following axioms of Kolmogorov.

(5) K1: $0 \leq [\![\phi]\!] \leq 1$;
K2: $[\![\phi \wedge \neg\phi]\!] = 0$ and $[\![\phi \vee \neg\phi]\!] = 1$;
K3: If $[\![\phi \wedge \psi]\!] = 0$, then $[\![\phi \vee \psi]\!] = [\![\phi]\!] + [\![\psi]\!]$.

In sections 2 and 3, I develop *Sample Logic*, a member of the family of Probabilistic Logics which is constructive, as close to linguistic data as possible, and obviates the challenges of other aforementioned logics. (I elaborate only upon its semantic part not upon its axiomatization which is left for future work.)

1.4 Linguistic Performance of Classical and Non-Classical Logics

In this preparatory section, I summarize (non-exhaustive) logical properties that play a role in linguistics and for which it is important to compare the performance of different logics.

A logic pertinent to linguistics should ensure the best possible engagement between linguistic data and the rigorousness of logical properties. It should be constructive (6a), should have complementary negation (6b), should be idempotent (6c), should be coherent (6d),[8] should connect conjunction and disjunction via de Morgan laws (6e+f),[9] should **differentiate**[10] between dependent and independent sentences (6g), should promote the material, residual and/or probabilistic conditional[11] in a manner that is sensible to linguistic data (6h/i/j), should exhibit the residual, material and/or probabilistic biconditional (6k/l/m), and should **distinguish** between conditionally dependent and independent sentences (6n).

I have compared seven logics on these properties, the findings of which are presented in Table 1. These seven logics are as follows: L_{Class} (Classical Bivalent Logic), L_3 (Kleene Trivalent

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[6] Lotfi Zadeh (2004), the founder of Fuzzy Logic, opines that Probabilistic Logic should be linked instead to Fuzzy Logic, but he did not present technical details.

[7] Adam (1998: 114) hedges at this problem by admitting two conditionals in Probability Logic, a probabilistic and a material conditional, stating: "Actually, it is only since Frege that the material conditional has become standard. The status of conditionals had long been a subject of controversy before that, as a remark attributed to the Hellenistic logician Callimachus attests: 'Even the crows on the rooftops are cawing over the question as to which conditionals are true.' (Mates, 1965: 203)."

[8] Ripley (2011) and Alxatib et al (2013) discuss speaker acceptability of contradictions. The empirical tests put before probands involve contractions with vague predicates (e.g. John is tall and not tall). Speakers sometimes accept contradictions because they telescope different contexts together or pursue certain goals in communication. Yet, I maintain that coherence is a crucial feature for any logic that attempts to model objective truth (see Kamp 1975).

[9] See Aloni (2016).

[10] The crucial question in (6g) is whether the logic draws a distinction between dependent and independent sentences that is useful in linguistics. Classical Logic, for example, treats all sentences as independent which is not sensible.

[11] Each of the logics listed promotes one conditional and one biconditional which I have marked in red font. Sometimes this (bi)conditional collapses with another (bi)conditional. In Łukasiewicz Fuzzy Logic, for example, the residual conditional is equivalent to the material conditional; in Classical Logic all three conditionals collapse.

Logic), Ł (Łukasiewicz Fuzzy Logic), L_{Min} (Gödel Fuzzy Logic) and L_{Prod} (Product Fuzzy Logic), L_{Adam} (Adam Probabilistic Logic), L_{Sam} (Sample Logic). For the semantics of the first five logics, refer to appendix, item B. The cells of Table 1 with bold underscored font highlight the (bi)conditional that is promoted by the logic in question.

(6)	Properties of $[\![\cdot]\!]:\mathrm{SENT} \to \{0,1\}\ or\ [0,1]$	L_{Class}	L_3	Ł	L_{Min}	L_{Prod}	L_{Adam}	L_{Sam}
a.	Constructive 2-step evaluation: Atomic/Complex	Yes	Yes	Yes	Yes	Yes	No	Yes
b.	Negation: $[\![\neg\phi]\!] = 1 - [\![\phi]\!]$	Yes	Yes	Yes	No	No	Yes	Yes
c.	Idempotency: $[\![\phi \wedge \phi]\!] = [\![\phi]\!]$	Yes	Yes	No	Yes	No	Yes	Yes
d.	Coherence: $[\![\phi \wedge \neg\phi]\!] = 0$	Yes	No	Yes	Yes	Yes	Yes	Yes
e.	First de Morgan law: $[\![\neg(\phi \wedge \psi)]\!] = [\![\neg\phi \vee \neg\psi]\!]$	Yes	Yes	Yes	Yes	Yes	Yes	Yes
f.	Second de Morgan law: $[\![\neg(\phi \vee \psi)]\!] = [\![\neg\phi \wedge \neg\psi]\!]$	Yes	Yes	Yes	Yes	Yes	Yes	Yes
g.	Independency: $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cdot [\![\psi]\!]$	No	No	No	No	No	Yes	Yes
h.	Material Conditional: $[\![\phi \to \psi]\!] = [\![\neg\phi \vee \psi]\!]$	**Yes**	**Yes**	Yes	No	No	No	No
i.	Residual Conditional: $[\![\phi \to \psi]\!] = 1 \Leftrightarrow [\![\phi]\!] \leq [\![\psi]\!]$	Yes	No	**Yes**	**Yes**	**Yes**	No	No
j.	Probabilistic Conditional: for $[\![\phi]\!] > 0 : [\![\phi \to \psi]\!] = \frac{[\![\phi \wedge \psi]\!]}{[\![\phi]\!]}$	Yes	No	No	No	No	**Yes**	**Yes**
k.	Material Biconditional: $[\![\phi \leftrightarrow \psi]\!] = [\![(\neg\phi \vee \psi) \wedge (\phi \vee \neg\psi)]\!]$	**Yes**	**Yes**	Yes	No	No	No	No
l.	Residual Biconditional: $[\![\phi \leftrightarrow \psi]\!] = [\![(\phi \to \psi) \wedge (\psi \to \phi)]\!]$	Yes	No	**Yes**	**Yes**	**Yes**	No	No
m.	Probabilistic Biconditional: $[\![\phi \leftrightarrow \psi]\!] = \begin{cases} [\![\psi]\!]/[\![\phi]\!] \text{ if } 0 < [\![\psi]\!] \leq [\![\phi]\!] \\ [\![\phi]\!]/[\![\psi]\!] \text{ if } 0 < [\![\phi]\!] < [\![\psi]\!] \end{cases}$	Yes	No	No	No	Yes	**Yes**	**Yes**
n.	Conditional Independency: $[\![\chi \to (\phi \wedge \psi)]\!] = [\![\chi \to \phi]\!] \cdot [\![\chi \to \psi]\!]$	No	No	No	No	No	Yes	Yes

Table 1: Logical properties in seven logics (with promoted bi/conditionals highlighted)

2. How sampling works on simple sentences

King Solomon's dictum appropriately explicates the mindset of Sample Logic:

> "What has been will be again, what has been done will be done again, there is nothing new under the sun" (Ecclesiastes 1:9);

as does the saying attributed to Mark Twain, alias Samuel Clemens (1835-1910):[12]

> "History doesn't repeat itself, but it often rhymes."

Both statements can be used to affirm the fundamental principle of experiments' repeatability. Owing to the fact that the population **Ω** of all events, states, and individuals is inaccessible and too large, we draw a sample **Σ** from **Ω** for each formula ϕ and form a property set **X** that comprises of those items in the sample that satisfy ϕ. Sometimes, it might be sufficient to just represent individuals. At other times, spelling out events would suffice.

[12] Mathematician and data scientist Catherine O'Neil (2016) argues in the New York Times best-seller *Weapons of Math Destruction* against the bias built in big data algorithms. Using a wide range of real-life examples, she cautions against the tendency of algorithms to extrapolate the future from past behavior even as to suggest that the unfettered use of AI (Artificial Intelligence) poses a threat to a democratic society. Her warnings impel us to build the samples on which Sample Logic is predicated with the greatest caution.

(7) *Sentence evaluation (Definition)*:
The truth degree or probability (**P**) of each sentence ϕ is the quotient of the property set size and the sample size: $[\![\phi]\!] = \mathbf{P} = \frac{|\mathbf{X}|}{|\mathbf{\Sigma}|} = \frac{card(\mathbf{X})}{card(\mathbf{\Sigma})}$.

Although the property sets may vary for different samples (and thus the truth degree **P**), the Central Limit Theorem in probability theory guarantees that the more samples we take, the closer **P** = |**X**|/|**Σ**| will get to the truth degree for the entire population (Butler 1985: 55; Gut 2005: 328). Thus, every sample and property set provide an approximation for the population.

In the following subsections, we review different sets of data which linguists previously had treated within several mutually incompatible logics. The aim of the subsequent discussion is to demonstrate that the same data can be dealt with in one unitary logic, Sample Logic.

2.1 Specific past events

Specific past events are evaluated in a singleton sample **Σ** that contains the event as a possibility and by a property set **X** that includes the event, if the event actually occurred, or is empty, if not. Therefore, the truth degree is either **P** = 1 or 0. The following example was a news piece reported by the *Jerusalem Post* on May, 7th of 2020 (Ben Harris 2020).

(8) Marilee Shapiro Asher contracted the Spanish Flu in 1918 (and the Coronavirus in 2020 and survived both).

2.2 Specific future events

The prediction of a specific future event **E** involves one or several context-induced predictors. A predictor of **E** is an event whose occurrence is correlated to the occurrence of **E**. (A correlated event can either be a causing event or just a 'symptom' of **E**.) The sample **Σ** to forecast a future event contains the predictor events, while the property set **X** includes past events of type **E** that are correlated to a predictor event. Consider examples in (9):

		Sample **Σ**	Property Set **X**	Truth Degree **P**
(9)	a. There will be a recession	{14 yield inversions}	{9 recessions};	64%;
	b. There will be a pandemic	{11 epidemics}	{9 pandemics}	82%;
	c. Bimbo will neither hike nor diminish its bakery business	{523 companies}	{214 companies}	41%
	d. Clorox will strongly increase sales of its disinfecting products	{349 companies}	{73 companies}	21%;
	e. The car rental company Hertz will file for bankruptcy	{216 companies}	{5 companies}	2%;
	f. Kimberley-Clark will greatly boost sales of its paper towels	{421 companies}	{80 companies}	19%

next year.

Suppose that the above predictions were made at the end of 2019. In (9a), financial analysts take the occurrence of yield curve inversions as a predictor of an upcoming recession, although the correlation is not absolute. A yield curve inversion is a scenario in which short-term debt instruments have higher yields than their long-term counterparts because investors apprehend a change in macro-economic conditions. The sample **Σ** of (9a) comprises of 14 past yield curve inversions since 1937, the timeframe of available comparative data (refer to appendix, item C). Nine of these 14 yield curve inversions were immediately followed by a recession (**X**). If a yield curve inversion occurred in the stated year, the truth degree of predicting *There will be a recession next year* is 64% (**P** = 9/14); if not it is 36% (**P** = 5/14). As a yield curve inversion did occur in 2019, prompting financial analysts to forecast a recession (Lewis 2019), it can be safely inferred that the occurrence of the Covid-19 pandemic has fulfilled this prediction.

Pandemics in (9b) are difficult to predict due to the lengthy chain of causation involved. The term 'pandemics' has no universally agreed upon definition, but 'wide geographic spread', 'high attack rates' and sometimes 'high death tolls' are typical features of a pandemic (Fauci et al 2009). Virologists keep track of hundreds of pathogens they find in bats, rats and fleas. As long as these viruses are confined within their pre-human hosts, the risk of a pandemic is zero. Starting from the day a pathogen is found in humans, its daily reproduction rate in the first month, when there is no lockdown to fetter its spread, provides the blueprint of a potential pandemic. Epidemiologists use the SIR Model (Hethcote 1989) to calculate the reproduction rate of a contagious disease and to predict its future course. The World Health Organization (WHO) published daily Covid-19 infections in Wuhan beginning from 02-Dec-2019, when the index case was discovered through the last day of 2019 (WHO 2020). From these data, which serve as a predictor of (9b), I approximate the reproduction ($\boldsymbol{\beta}$) and recovery ($\boldsymbol{\gamma}$) rates of Covid-19 (for details, refer to appendix, item D). Using entries of Byrne (2008)'s *Encyclopedia of Pandemics and Plagues*, I assume that 11 historical epidemics ($\boldsymbol{\Sigma}$) show reproduction and recovery rates similar to Covid-19. The property set (**X**) consists of nine epidemics in the sample that went on to be known as historical pandemics. The forecast of *There will be a pandemic next year* made with the knowledge available at the end of 2019 is therefore true to the degree 82% (**P** = 9/11).

Examples (9c-f) are forecasts about the economic performance of four globally operating companies. Bimbo is a Mexico-based maker of industrial bakery goods, whereas the US-company Clorox manufactures disinfecting products. Similarly, Hertz is a car rental company with headquarters in Florida, and the US-corporation Kimberley-Clark produces consumer tissues such as paper towels. Statements about the future performance of companies involve macroeconomic and microeconomic predictors. From a macroeconomic perspective, bakery products (Bimbo)[13] and consumer tissues (Kimberley-Clark)[14] are essential consumer goods, the demand of which thrives during the period of a ubiquitous disaster. However, both these industries appear to be indifferent to economic recessions unless when they co-occur with a disaster. The outbreak of a viral or bacterial epidemic is a macroeconomic predictor for good performance of Clorox.[15] In a similar manner, the occurrence of an economic recession is a macroeconomic indicator for the bankruptcy of Hertz.[16] Furthermore, the main microeconomic predictor for the above statements (9c-f) is a change in profitability for the company in question. For example, an increase in profit margins allows the company to reduce the price of its products in the following year which, in turn, may raise its sales and market share. The sample and property set for the above examples involve companies and their economic activities carried out in the previous years. For instance, the sample $\boldsymbol{\Sigma}$ of (9d) comprises of Clorox's partners and competitors[17] that did business during a past epidemic (macroeconomic predictor) or that were as profitable as Clorox in 2019 (microeconomic predictor), altogether 349 companies. The property set **X** includes companies of the sample that witnessed a 'strong' increase (>9%) of their sales, which was the case with 73 companies. The foretelling of *Clorox will strongly increase sales of its disinfecting products* is therefore true to the degree 21% (**P** = 73/349). For more details of how the truth degrees of (9c-f) are calculated, refer to appendix, item E.

[13] The vice-president of Grupo Bimbo (annual sales: 15 billion US$), Jorge Zárate, reports a strong boost of demand in the first quarter of 2020 due to the outbreak of Covid-19 (Spencer 2020).

[14] According to Niki Edwards, senior lecturer at the School of Public Health and Social Work, Queensland University of Technology, Australia, people stockpile toilet paper and other consumer tissues during a period of disaster in order to compensate for a loss of control (Lucy 2020).

[15] In addition to epidemics, disasters such as earthquakes that result in widespread injuries may also cause a surge in demand for disinfecting products.

[16] A recession, especially if it is severe, results in diminished economic activity. As people rent cars for their economic activities, a recession represents a financial risk for car rental companies. Hertz Global Holdings announced bankruptcy on 22-May-2020 (Kelly 2020) indirectly due to a recession and more directly because of the lockdown related to the Covid-19 pandemic, which brought outdoor activities to a complete halt.

[17] Company A is a partner of Company B if consumers buy A's products together with B's products. A is a competitor of B if consumers either buy A's products or B's products. For a mutual partnership or mutual competition to be meaningful, the affected sales must lie above a certain threshold, say 1% of yearly sales of A and B.

2.3 Modal events

Scholars have traditionally interpreted modal expressions by bivalent truth and possible worlds (Groenedijk and Stokhof 1975; Kratzer 1977, 1981, 1991; Lewis 1981).[18] Any of (10a-f) below is true in a world *w* if the clause without the auxiliary verb is true in *all* or *some* world *w′* accessible from *w*. Accessibility relations are derived from conversational backgrounds, which should be understood as sets of propositions that capture a context's properties and which can be glossed by expressions such as *in view of what is known* (epistemic) or *in view of the duties* (deontic). For example, (10a) is true in a world *w* if the clause without the auxiliary verb is true in every world *w′* wherein everything that is known in *w* is true. As an amendment to an internal paradox in the formalization,[19] Kratzer (1981, 1991) introduced *Graded Possible World Semantics* (GPWS) where accessibility relations are derived from **two** conversational backgrounds, one is context-independent and the other context-dependent. The second conversational background imposes a partial order "<" on the worlds defined by the first. Necessary truth in *w* is evaluated in ideal worlds (or minimal worlds in the sense of "<") accessible from *w*. Besides modal auxiliaries, GPWS was also applied to the formalization of the English progressive aspect -*ing* (Portner 1998).

(10) John		Sample **Σ**	Property Set **X**	Truth Degree **P**
a.	must (=will certainly)	{187 teachers}	{138 teachers}	74%
b.	might	{1 teacher}	{1 teacher}	100%
c.	will	{181 teachers}	{138 teachers}	76%
d.	must (=is obliged to)	{1 duty}; {208 followers}	{1/0 duty}; {120 positive}	100%; 0%; 72%
e.	can	{1 duty/ban}; {208 followers}	{0/0}; {80 indeterminate}	0%; 0%; 24%
f.	must not	{1 ban}; {208 followers}	{1/0 ban}; {8 negative}	0%; 100%; 4%

get vaccinated against Covid-19 within three months.

The first three of these modal statements are epistemic, the last three are deontic expressions. I will pursue a different analysis of modal expressions. In Sample Logic, epistemic statements such as the one in (10a-c) are predictions about the future, whereas deontic statements in (10d-f) are not. The truth of an obligation does not lie in the probability of its future fulfilment, but in the truth of its existence at the time of speaking.

Suppose that the statements (10a-f) were made at the end of 2020. Global rollout of Covid-19 vaccines started during December 2020 allowing for a certain amount of comparative data.[20] The main predictors for Covid-19 vaccination in (10a-c) are availability of the vaccine, access given to a professional category and personal willingness. During 2020, John's country of residence signed contracts with three manufacturers of mRNA-vaccines who agreed to deliver daily doses. As a teacher, John is given preferential access to the vaccine, although he is worried about adverse effects. Before the rollout, his personal details and opinions were registered in an applicant database. Health officials promised to deliver 35.000 doses of vaccines to his district and to offer vaccinations to 158 instructors out of a teacher population of 187 before the end of March 2021. (The 158 instructors were chosen according to a distribution key of the Government.) Thirty-one teachers with the same predictor profile as John received an offer even before the end of 2020, but only twenty-seven were actually vaccinated. The sample **Σ** for (10a) comprises of the 187 teachers in John's district, while the property set **X** contains 138 (≈ 158 x 27/31) teachers,

[18] Possible-world semantics was put on a formal basis in the seminal papers of Samuel Kripke (1959, 1963) who also introduced the notion of an accessibility relation *R*.

[19] This paradox, called *Samaritan Paradox*, is one of Prior's paradoxes of derived obligation (Prior 1954).

[20] O'Neil (2016)'s main criticism against big data algorithms is their attempt to predict future behavior on the basis of past behavior in cases where the predicted event, because of its uniqueness, does not follow a genuine template. In spite of this difficulty, I attempt to propose a probability for *every* future event so that Sample Logic is not a model with "evaluation gaps".

the extrapolated number of teachers with John's predictor profile who get vaccinated by the end of March 2021. The epistemic auxiliaries *must*, *might* and *will* affect the truth value of the prediction in different ways. *Must* and *might* act as universal (respectively, existential) quantifier over the sample of 187 teachers.

(11) a. *John* ***must*** *get vaccinated within three months* in (10a) is true, if and only if **all** teachers with John's predictor profile get vaccinated within three months.

Since only 138 teachers with John's predictor profile were expected to get vaccinated, the truth degree of (10a) is lower with 74% (**P** = 138/187).

b. *John* ***might*** *get vaccinated within three months* in (10b) is true, if and only if **there is** someone with John's predictor profile who gets vaccinated within three months.

Since there is not only one but 138 teachers with John's predictor profile who were expected to get vaccinated, the truth degree is 100% (**P** = 1/1). The unmarked way of stating a prediction involves the auxiliary *will* (or morphological future tense in languages like French). In a context where *will* is not contrasted with *must*, like in (9), *will*-predictions are approximations of *must*-predictions. If they are contrasted, as in (10), we understand *will* as an *almost*-universal quantifier.

c. *John* ***will*** *get vaccinated within three months* in (10c) is true, if and only if **almost all** (97%) teachers with John's predictor profile get vaccinated within three months.

Thus, the sample of (10c) takes six predictor profiles (ca. 3%) that are not linked to a vaccination off the base sample, shrinking to 181 predictor profiles. The truth degree of *John will get vaccinated within three months* is 76% (**P** = 138/181).

The rating of external deontic forces in (10d-f) first depends on legal obligation. If the law of a country authorizes the government to mandate vaccinations and if the government decides to do so, [21] then (10d) has truth degree one, (10e) and (10f) truth degree zero. If John's medical history[22] or his membership to a religious group[23] forbids him to take the vaccine, then (10d) and (10e) are false and (10f) true. Apart from legal obligation, medical condition and religious belief, (10d-f) are evaluated by the degree of social acceptance (which in turn is shaped by mass media). Suppose that followers on Twitter signify people John has social relations with. John surveys his followers' stance on Covid-19 vaccinations by giving opportunity to provide feedback with the following smileys: ☺ positive; 😐 indeterminate; ☹ negative. These choices are taken as proxies for obligation, permission and prohibition; 150 of his 208 followers[24] are positive, 50 are indeterminate and eight are negative. The respective truth degrees for (10d-f) can be derived accordingly.

2.4 Generalized Quantifiers

Barwise and Cooper (1981) pioneered the view of noun phrases (*John*, *all scholars*) and noun determiners (*all*) as quantifiers, referred to as *generalized quantifiers*. Van Benthem (1984), Keenan and Stavi (1986), Keenan (1996), Keenan and Westerståhl (1997) further developed and streamlined the theory. In this subsection, I interpret generalized quantifiers in Sample Logic.

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[21] Covid-19 vaccinations are mandatory for healthcare workers in Italy since April 2021.

[22] In the event of severe allergic reactions and certain autoimmune diseases, Covid-19 vaccinations are harmful.

[23] Between 1931 and 1952, the Jehovah's Witnesses opposed vaccinations believed to be a violation of Leviticus 17: 10-11, but lifted the ban after 1952. The equation of mRNA vaccines with system software by pharmaceutical corporations (e.g. Moderna 2021) made some religious groups suspicious of a scheme leading up to the 'mark of the beast' (Revelation 13).

[24] According to the analytics software *Beevolve*, the average number of followers per user on Twitter was 208 in 2012.

Suppose that a group of ornithologists earmarked a district in a nature reserve and put tiny infrared cameras on all n owls they found there (including m adult owls and n-m baby owls). The ornithologists found evidence of a very large mouse population in that district. They estimated that the upper limit of possible mouse-hunting events for 180 days was q per owl because of various constraints. During the three-month trial, the cameras documented all hunting events, including the type of animals caught. At the end, the ornithologists counted the actual number of hunted mice, successful or not, as $u_k \leq q$, for each owl k. They organized the owl population A in accordance with the number of chased mice: $A = \{(k, u_k) \mid \forall i < k: u_i \leq u_k\}$. The ornithologist had to decide which owl to be classified as prey bird since no owl hunted the maximal number of 180 mice. For example, the baby owl did not hunt a single mouse. Thus, they put in place the threshold of r mice below which they considered an owl not to be a hunter. Given this threshold, let denote $v_k = \mathbf{min}\ \{u_k, r\}$ and V_k = {hunting events of owl k cropped by r}. With these data, full truth conditions for the most common generalized quantifiers are spelled out in (12).

(12)	Conditions for Full Truth
a. Every owl always hunts mice.	$\forall i$: $u_i = q$.
b. Every owl hunts mice.	$\forall i$: $v_i \geq r$.
c. An owl hunts mice.	$\forall i$: $v_i \geq r$ (i is an adult-owl).
d. At least k owls hunt mice.	$\exists i_1 \ldots \exists i_k$: $i_1 \neq \ldots \neq i_k \wedge v_{i_1} \geq r \wedge \ldots \wedge v_{i_k} \geq r$.
e. At most k owls hunt mice.	$\exists i_1 \ldots \exists i_{n-k}$: $i_1 \neq \ldots \neq i_{n-k} \wedge v_{i_1} < r \wedge \ldots \wedge v_{i_{n-k}} < r$.
f. Only k owls hunt mice.	$\exists i_1 \ldots \exists i_n$: $i_1 \neq \ldots \neq i_n \wedge v_{i_1} < r \wedge \ldots \wedge v_{i_{n-k}} < r \wedge v_{i_{n-k+1}} \geq r \wedge \ldots \wedge v_{i_n} \geq r$.
g. Some owls hunt mice.	At least ***one*** owl hunts mice.
h. Most owls hunt mice.	At least **half** of the owls hunt mice.

These statements may be less than fully true. In order to assign smaller truth degrees, we define samples and property sets based on the truth conditions in (12). An example of a population of 27 owls with their respective hunting events is provided (refer to appendix, item F, Table 8).

Universal quantification in the first three examples (12a-c) can be understood as manifold conjunctions. The sample and property sets are set products of possible and actual hunting events over A, the owl population. The corresponding truth degree is a quotient of two numerical products. The truth degree of *every owl (always) hunts mice* in (12a-b) equals zero due to the non-hunting baby owls in the population. Generic statements like in (12c) are 'law-like' (Kadmon and Landman 1993: 357) while simultaneously allowing exceptions. The m adults of the owl population that do hunt to some degree are the rule, while the n-m non-hunting baby owls are the exception. By replacing n with m, the sample and property set for the generic quantifier in (12c) are almost the same as for the universal quantifier in (12b). In the example (appendix item F, Table 8), the truth degree of *an owl hunts mice* is 16%.

The truth degree of *at least k owls hunt mice* in (12d) is the number of those k best performing owls that hunt at least r mice divided by k. In the example (Table 8), nineteen owls are prey birds as defined by the ornithologists. The truth degree of *at least 21 owls hunt mice* is thus 90% (**P** = 19/21).

The statement *at most k owls hunt mice* is true if *at least n-k owls do not hunt mice* is true. The truth degree of (12e) is thus the number of those n-k worst performing birds that hunt less than r mice divided by n-k. In Table 8, eight of 27 owls do not count as hunters. This is why the truth degree of *at most 11 owls hunt mice* is 50% (**P** = 8/16). In (12f), *only k owls hunt mice* is true if the lower n-k owls of the population do hunt less than r mice and the upper k owls do hunt more than r-1 mice. In Table 8, this is only true if $k = 19$, and otherwise false. In concordance with the theory of generalized quantifiers, *some* in (12g) is interpreted as *at least one*, and *most* in (12h) as *at least half of*. Both truth degrees are one in the example.

(13)	Sample **Σ**	Property Set **X**	Truth Degree **P**
a. Every owl always hunts mice.	$\{q\ hunts\}^n$	$\prod_{k=1}^{n}\{u_k\ hunts\}$	$\prod_{k=1}^{n}u_k \big/ q^n = 0\%$
b. Every owl hunts mice.	$\{r\ hunts\}^n$	$\prod_{k=1}^{n}V_k$	$\prod_{k=1}^{n}v_k \big/ r^n = 0\%$
c. An owl hunts mice.	$\{r\ hunts\}^m$	$\prod_{k=1}^{m}V_k$	$\prod_{k=1}^{m}v_k \big/ r^m = 16\%$
d. At least *k* owls hunt mice.	$A_k = \left\{ \begin{matrix} (n-k+1, u_{n-k+1}), \ldots \\ \ldots, (n, u_n) \in A \end{matrix} \right\}$	$B_k = \left\{ \begin{matrix} (i,u_i) \in A_k \mid \\ u_i \geq r \end{matrix} \right\}$	$\frac{\lvert B_k \rvert}{\lvert A_k \rvert}\left(90\% = \frac{19}{21}, k = 21\right)$
e. At most *k* owls hunt mice.	$C_k = \left\{ \begin{matrix} (1, u_1), \ldots \\ \ldots, (n-k, u_{n-k}) \in A \end{matrix} \right\}$	$D_k = \left\{ \begin{matrix} (i,u_i) \in C_k \mid \\ u_i < r \end{matrix} \right\}$	$\frac{\lvert D_k \rvert}{\lvert C_k \rvert}\left(50\% = \frac{8}{16}, k = 11\right).$
f. Only *k* owls hunt mice.	$E_k = \{(n-k, u_{n-k}) \in A\}$	$F_k = \left\{ \begin{matrix} (i,u_i) \in E_k \mid \\ u_i < r \wedge u_{i+1} \geq r \end{matrix} \right\}$	$\frac{\lvert F_k \rvert}{\lvert E_k \rvert}\left(\begin{matrix} 100\% \text{ if } k = 19 \\ 0\% \text{ if } k \neq 19 \end{matrix} \right).$
g. Some owls hunt mice.	$A_1 = \{(n, u_n) \in A\}$	$B_1 = \left\{ \begin{matrix} (n,u_n) \in A_1 \mid \\ u_n \geq r \end{matrix} \right\}$	$\frac{\lvert B_1 \rvert}{\lvert A_1 \rvert} = 100\% = \frac{1}{1}.$
h. Most owls hunt mice. (with *k* = round{*n*/2})	$A_k = \left\{ \begin{matrix} (n-k+1, u_{n-k+1}), \ldots \\ \ldots, (n, u_n) \in A \end{matrix} \right\}$	$B_k = \left\{ \begin{matrix} (i,u_i) \in A_k \mid \\ u_i \geq r \end{matrix} \right\}$	$\frac{\lvert B_k \rvert}{\lvert A_k \rvert} = 100\% = \frac{14}{14}.$

2.5 Gradable States

There exist two distinct analyses of gradable adjective meaning. The first assigns a proper ontology to degrees (at par with times, possible worlds, individuals and truth values) and evaluates adjectives as partial functions from individuals to degrees (Cresswell 1976; Hellan 1981; Kennedy 1999, 2001, 2007; Seuren 1978; von Stechow 1984). In this line, degrees are either employed as unanalyzed primitive or further broken down as equivalence classes of individuals (Cresswell 1976) or as convex sets and particularly as intervals of the real number space (Seuren 1978; von Stechow 1984; Kennedy 2001). The second approach interprets gradable adjectives as partial functions from a (context-dependent) comparison class to truth values (Fine 1975; Gerner 2007; McConnell-Ginet 1973; Kamp 1975; Klein 1980; Pinkal 1995). I adopt this analysis since comparison classes are directly amenable as samples.

In Sample Logic, gradable adjectives, gradable antonymic pairs, as well as comparative and superlative forms, are interpreted by comparison classes (or samples) induced by the context. Pairwise comparison between the subject and each member of the comparison class yields three sets, the sets of positive, negative, and inconclusive comparisons. The truth degree is the size ratio of the set of positive comparisons and the sample. The statements below are drawn from the 30 teams of the NBA, the *National Basketball Association*, with its 497 registered players.[25]

			Local **P**	Global **P**
(14)	a.	Tyson Chandler is the tallest.	15/16	465/496
	b.	Tyson Chandler is the smallest.	0/16	9/496
	c.	Troy Daniels is tall.	1/7	101/247
	d.	Troy Daniels is small.	8/8	247/247
	e.	Tacko Fall is taller (smaller) than Chris Clemons.	---	1/1 (0/1)
	f.	The players of Dallas Mavericks are taller than those of Utah Jazz.	---	162/289
	g.	The players of Chicago Bulls are the tallest.	---	4152/8160
	h.	The players of Dallas Mavericks are tall.	---	2816/4080

[25] Data were taken from the official website of the *National Basketball Association* (https://www.nba.com/) in April 2020.

The superlative in (14a) can be evaluated in the sample of the 17 Houston Rockets players Tyson Chandler is a member of, or in the sample of all 497 NBA players. Tyson has the maximal length of 213 cm (7 ft) in his own team, but shares this height with co-player Isaiah Hartenstein. The comparison class comprises of the Houston Rockets players without Tyson Chandler, while the set of positive comparisons includes 15 players. The truth degree of *Tyson Chandler is the tallest* is 94% (**P** = 15/16). Now, if all NBA players are taken into consideration, then the truth degree is 94% (**P** = 465/496) (465 players are smaller than Tyson Chandler) which coincidentally equals the previous truth degree. In (14b), the set of positive comparisons (for small) is empty for the Houston Rockets team, but counts nine players for the entire NBA (nine NBA players are taller than Tyson Chandler). The truth degree of *Tyson Chandler is the smallest* is thus 0% (**P** = 0/16), respectively 2% (**P** = 9/496). [26]

In the comparison-class approach, (14c), *Troy Daniels is tall* is true, if Troy happens to be taller than the *average degree* (Bartsch and Vennemann 1973). There are three measures of central tendency in statistics: the mean,[27] the median[28], and the mode.[29] If the heights are normally distributed, all three measures coincide (mode = median = mean). However, if the distribution is positively skewed, then mean > median > mode; if it is negatively skewed, then mode > median > mean. A problem thus arises as to which average measure should be selected in the evaluation of (14c). Kennedy (2007) alludes to the same problem when discussing the truth of *A rent of $725 is expensive for an apartment on this street*, assuming that the median rent for apartments on the street is $700 and that there are a few apartments with rents significantly higher than $725. He concludes that in this case, one would be reluctant to judge this sentence as true. Owing to these few very expensive apartments, Kennedy seems to refer to a positively skewed rent distribution. If we take the mean rent (say at $750) rather than the median rent ($700), the statement comes out as false (as one would expect). More generally, we can take whatever measure has a greater value (i.e. the mean or the median).

In Troy Daniels's team, the Denver Nuggets, the mean (201.87cm) is slightly higher than the median (201.50cm), while the median (199.80cm) is a bit higher than the mean (199.18cm) in the entire NBA. However, these differences do not play a role in this particular example; we can take either the mean or the median value. Thus, the sample of (14c) consists of the lower half of Denver Nuggets players (resp. NBA players) without Troy. With these specifications, the sample consists of seven DN players (resp. 247 NBA players). The set of positive comparisons comprises of one DN player since Troy Daniels, whose height is 193 cm (6 ft 4 in), is only taller than one other player (resp. taller than 101 NBA players). The truth degree of *Troy Daniels is tall* is thus 14% (**P** = 1/7) (resp. 41% or **P** = 101/247).

The sample for the antonymic adjective *small* in (14d) uses the upper half of the DN (/NBA) players without Troy, which counts eight (/247) players. The set of positive comparisons consists of those DN (/NBA) players who are taller than the medium height 201.5 cm (/199.8 cm). Given Troy's height of 193 cm, this set counts eight (/247) elements. Therefore, the statement *Troy Daniels is small* is maximally true with true degree *one* in both contexts.

Comparative predicates like *taller*/*smaller* in (14e) employ as sample, the singleton set containing the standard of comparison, Chris Clemons. The set of positive comparisons only includes Chris Clemons if Tacko Fall is actually taller/smaller than Chris Clemons and is empty, if not. Given that Chris Clemons of Houston Rockets is the smallest NBA player with 175 cm (5 ft 9 in) and Tacko Fall of Boston Celtics the tallest NBA player with 226 cm (7 ft 5 in), the

[26] The fact that superlative or comparative sentences are evaluated with intermediate truth values does not mean that these sentences can be graded or modified by syntactic modifiers (*very*, *slightly* etc.). Truth and grammaticality are two separate notions.

[27] The mean is defined as the sum of measurements divided by the number of observations.

[28] The median is the value that has an equal number of observations above and below.

[29] The mode is the value with the highest frequency (or the highest number of observations). Due to the relatively small team heights, the mode is not a useful measure of central tendency here.

statement *Tacko Fall is taller than Chris Clemons* has truth degree one and *Tacko Fall is smaller than Chris Clemons* the truth degree zero.

A comparison of two teams like in (14f) may have many truth degrees depending on how many players of the first team surpass how many players of the second team. If $\mathbf{\Sigma}_1$ denotes the Dallas Mavericks and $\mathbf{\Sigma}_2$ the Utah Jazz, then the sample for (14f) represents the set of all possible comparisons, which is $\mathbf{\Sigma} = \mathbf{\Sigma}_1 \times \mathbf{\Sigma}_2$. The set of positive comparisons consists of $\mathbf{X} = \{(x_1, x_2) \mid x_1 > x_2\}$ and the statement *The players of Dallas Mavericks are taller than the players of Utah Jazz* is true to the degree 56% (**P** = 162/289). The size of **X** can be established by counting the number of positive comparisons (refer to appendix, item G). If all DM players were taller than all UJ players, the set X would be equal to $\mathbf{\Sigma}$ and the truth degree one; if, on the other hand, all DM players were smaller than all UJ players, the set **X** would be empty and the truth degree zero. Therefore, the reality lies between these two extremes.

If a team like the Chicago Bulls in (14g) is not compared to one other particular team but to all teams of the NBA, then the sample consists of all possible comparisons $\mathbf{\Sigma} = \mathbf{\Sigma}_1 \times \mathbf{\Sigma}_2$ between Chicago Bulls players ($\mathbf{\Sigma}_1$) and other NBA players pooled together ($\mathbf{\Sigma}_2 = \bigcup \mathbf{\Sigma}_k \setminus \mathbf{\Sigma}_1$). Since the number of NBA players without Chicago Bulls is 480, the sample size is 8160 = 17×480. With the set of positive comparisons being defined as $\mathbf{X} = \{(x_1, x_2) \in \mathbf{\Sigma} \mid x_1 > x_2\}$, we can count the size of **X** as the number 4152 (refer to appendix, item H). Consequently, the statement *The players of Chicago Bulls are the tallest* is true to the degree of 51% (**P** = 4152/8160).

In the last example (14h), the players of the Dallas Mavericks only need to be compared to the lower half of the NBA players without Dallas Mavericks, which amounts to 240 players. In this sample ($\mathbf{\Sigma} = \mathbf{\Sigma}_1 \times \mathbf{\Sigma}_2$), we can count the set of positive comparisons **X** to have 2816 elements (refer to appendix, item I). Stating *The players of Dallas Mavericks are tall* carries a 69% (**P** = 2816/4080) kernel of truth.

Positive adjectives can have borderline cases and thus give rise to borderline contradictions (section 1.1) and to the sorites paradox (Kennedy 2007; Sorensen 2018). The sorites paradox has fascinated philosophers because it attempts to demonstrate vague properties by using the mathematical proof technique of *complete induction*. The sorites paradox falsely suggests that the induction step is valid for all increments although it is not. (To put it differently, vague predicates only allow partial not complete induction.)

(15)	a.	Borderline Case:	Jimmy Butler of Miami Heat is tall.
	b.	Borderline Contradiction:	Jimmy Butler of Miami Heat is tall and not tall.
(16)	a.	Sorites Paradox Base Step:	A 175 cm tall NBA player is small.
	b.	Induction Step:	If an *n* cm tall NBA player is small, then an *n*+1 cm tall NBA player is also small.
	c.	Conclusion:	Therefore, a 226 cm tall NBA player is small.

Since Jimmy Butler's height of 199,80 cm is the medium height of all 497 NBA players, it implies that he is taller than 248 players and also smaller than 248 players. The problems of borderline cases and borderline contradiction are settled by assigning the truth degree 50% to (15a) and the truth degree 0% to (15b). The Sorites Paradox is solved because the induction step becomes invalid for n = 199 cm.

3. Sample Logic and Complex Sentences

We assume a population $\mathbf{\Omega}$ of events, states, and entities, from which we draw samples $\mathbf{\Sigma}$. In general, a sample comprises of *n*-tuples of events with variable length *n*. Let ϕ and ψ be two formulas with samples $\mathbf{\Sigma}_\phi$, $\mathbf{\Sigma}_\psi$ and property sets $\mathbf{X}_\phi$, $\mathbf{X}_\psi$.

3.1 Negation, Conjunction, Disjunction

(17) *Definition of Negation:*

The negation $\neg\phi$ is interpreted by the size ratio of the property set $\mathbf{X}_{\neg\phi} = \mathbf{\Sigma}_{\phi} \setminus \mathbf{X}_{\phi}$ and the sample $\mathbf{\Sigma}_{\neg\phi} = \mathbf{\Sigma}_{\phi}$: $[\![\neg\phi]\!] = \frac{|X_{\neg\phi}|}{|\Sigma_{\neg\phi}|} = \frac{|\Sigma_{\phi} \setminus X_{\phi}|}{|\Sigma_{\phi}|}$.

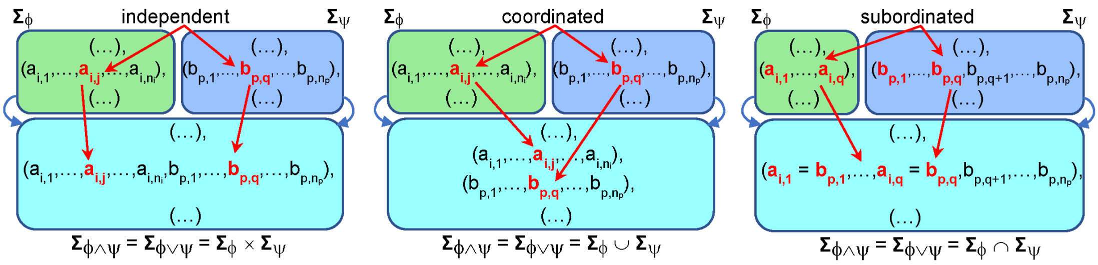
In section 2, most simple sentences are not genuine atomic propositions; instead, they represent hidden conjunctions or disjunctions. Atomic propositions can be defined in Sample Logic as those simple sentences that have a singleton sample. To begin with, I illustrate for three types of atomic propositions, partly quoted from section 1.1, how conjunction is defined in Sample Logic. Example (18a) conjoins two independent propositions. Their pooled sample and property sets are defined as Cartesian products. Examples (18b) and (18c) combine two pairs of dependent propositions. The difference between the two examples is that their component samples have empty intersection in (18b) and are identical in (18c). Accordingly, the property set are defined as union in (18b), and as intersection in (18c).[30]

(18)	Atomic propositions (*Example*)	Sample $\mathbf{\Sigma}_{\phi\wedge\psi}$ and Property Set $\mathbf{X}_{\phi\wedge\psi}$	**P**
a.	ϕ: Nicolas Batum is taller than Cody Zeller and ψ: Joe Chealey is taller than Terry Rozier.	$\mathbf{\Sigma}_{\phi\wedge\psi}$ = {(Batum, Zeller)} × {(Chealey, Rozier)} $\mathbf{X}_{\phi\wedge\psi}$ = ∅ × {(Chealey, Rozier)} = ∅	0%
b.	ϕ: Joe Chealey is taller than Caleb Martin and ψ: Terry Rozier.	$\mathbf{\Sigma}_{\phi\wedge\psi}$ = {(Chealey, Martin)} ∪ {(Chealey, Rozier)} $\mathbf{X}_{\phi\wedge\psi}$ = ∅ ∪ {(Chealey, Rozier)}	50%
c.	ϕ: Kobi Simmons is not taller and ψ: not smaller than Caleb Martin.	$\mathbf{\Sigma}_{\phi\wedge\psi}$ = {(Simmons, Martin)} ∪ {(Simmons, Martin)} $\mathbf{X}_{\phi\wedge\psi}$ = {(Simmons, Martin)} ∩ {(Simmons, Martin)}	100%

We now turn to the general conjunction of two propositions. Two independent sentences have samples where no item in one sample is correlated to any item in the other sample. Sentences are dependent when their semantic relation is one of subordination or of coordination. Both sentences are on equal terms in a coordination relation and exhibit samples where at least one item in the first sample is corelated to another item in the second sample. Meanwhile, one sentence provides the frame for the other in a subordinated relation. The sample of a subordinated sentence contains at least one *n*-tuple of items in which one *k*-tuple of the other sample is woven ($k < n$). Uncorrelated samples are pooled as Cartesian products, coordinated samples as unions, and subordinated samples as (abstract) intersections.

(19) *Definition of Conjunction and Disjunction:*

a. Pooling



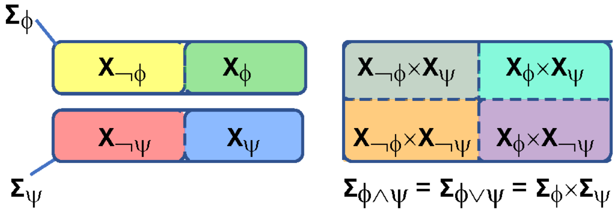
Taking these pooling principles into consideration, the conjunctive and disjunctive samples and property sets are defined as follows:

[30] The names in the samples are provided as identifiers of the body height: Cody Zeller 213 cm; Nicolas Batum 206 cm; Kobi Simmons 196 cm; Caleb Martin 196 cm; Joe Chealey 193 cm; and Terry Rozier 185 cm.

b. **Σ_ϕ and Σ_ψ are independent (uncorrelated)**

Property Sets: $X_{\phi\wedge\psi} = X_\phi \times X_\psi$ and
$X_{\phi\vee\psi} = (X_\phi \times X_\psi) \cup (X_{\neg\phi} \times X_\psi) \cup (X_\phi \times X_{\neg\psi})$

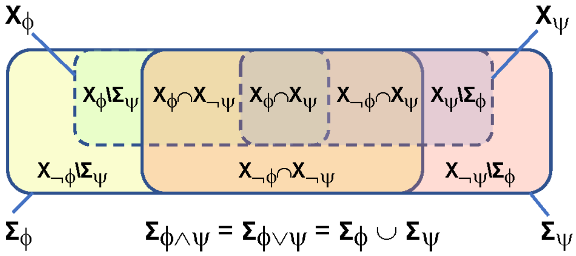
Sample: $\Sigma_{\phi\wedge\psi} = \Sigma_{\phi\vee\psi} = \Sigma_\phi \times \Sigma_\psi$



c. **Σ_ϕ and Σ_ψ are coordinated**

Property Sets: $X_{\phi\wedge\psi} = (X_\phi \cap X_\psi) \cup (X_\phi \setminus \Sigma_\psi) \cup (X_\psi \setminus \Sigma_\phi)$ and
$X_{\phi\vee\psi} = X_\phi \cup X_\psi$

Sample: $\Sigma_{\phi\wedge\psi} = \Sigma_{\phi\vee\psi} = \Sigma_\phi \cup \Sigma_\psi$



Subcases	(i) $\Sigma_\phi \cap \Sigma_\psi = \varnothing$ *"Interdependency"*	(ii) $\Sigma_\phi \subset \Sigma_\psi$ and $\Sigma_\phi \neq \Sigma_\psi$ *"Skewed dependency"*	(iii) $\Sigma_\phi = \Sigma_\psi$ *"Interrelation"*
Property Sets: $X_{\phi\wedge\psi}$ =	$X_\phi \cup X_\psi$;	$(X_\phi \cap X_\psi) \cup (X_\psi \setminus \Sigma_\phi)$;	$X_\phi \cap X_\psi$;
$X_{\phi\vee\psi}$ =	$X_\phi \cup X_\psi$;	$X_\phi \cup X_\psi$;	$X_\phi \cup X_\psi$;
Sample: $\Sigma_{\phi\wedge\psi} = \Sigma_{\phi\vee\psi}$ =	$\Sigma_\phi \cup \Sigma_\psi$	Σ_ψ	$\Sigma_\phi = \Sigma_\psi$

d. **Σ_ψ is subordinated to Σ_ϕ**

Property Sets: $X_{\phi\wedge\psi} = \{(b_1,\ldots b_n) \in X_\psi \mid \exists(a_1,\ldots,a_i) \in X_\phi: a_1 = b_1,\ldots, a_i = b_i$, and $\forall(c_1,\ldots,c_j) \in X_{\neg\phi}: c_1 \neq b_1$ or…or $c_j \neq b_j\}$

$X_{\phi\vee\psi} = X_{\phi\wedge\psi} \cup X_{\neg\phi\wedge\psi} \cup X_{\phi\wedge\neg\psi}$

Sample: $\Sigma_{\phi\wedge\psi} = \Sigma_{\phi\vee\psi} = \{(b_1,\ldots b_n) \in \Sigma_\psi \mid \exists(a_1,\ldots,a_i) \in \Sigma_\phi: a_1 = b_1,\ldots, a_i = b_i\}$

e. The truth degrees of conjunction and disjunction are defined as follows:

$$[\![\phi \wedge \psi]\!] = \frac{|X_{\phi\wedge\psi}|}{|\Sigma_{\phi\wedge\psi}|} \text{ and } [\![\phi \vee \psi]\!] = \frac{|X_{\phi\vee\psi}|}{|\Sigma_{\phi\vee\psi}|}.$$

With these specifications, it is possible to show that $[\![\cdot]\!]$: SENT $\rightarrow$ [0,1] is a probabilistic evaluation that satisfies a slightly weakened version of the Kolmogorov axioms. Considering the fact that we are dealing with samples and not with populations, it is not possible to demonstrate K3 for dependent sentences only for independent sentences (K3')..[31] However, we can demonstrate the inequality of axiom K4 for independent and coordinated sentences (for proof refer to appendix, item J):

(20) K1 $0 \leq [\![\phi]\!] \leq 1$;
K2 $[\![\phi \wedge \neg\phi]\!] = 0$ and $[\![\phi \vee \neg\phi]\!] = 1$;
K3' If ϕ and ψ are independent and $[\![\phi \wedge \psi]\!] = 0$, then $[\![\phi \vee \psi]\!] = [\![\phi]\!] + [\![\psi]\!]$.
K4 If ϕ and ψ are independent or coordinated, then $[\![\phi \vee \psi]\!] \leq [\![\phi]\!] + [\![\psi]\!]$.

[31] Besides independent propositions, K3 also holds for interrelations, see (19c-iii).

Furthermore, it follows that $[\![\neg\phi]\!] = 1 - [\![\phi]\!]$, that $[\![\cdot]\!]$ is idempotent and coherent, and that the two de Morgan laws hold (for proof, refer to appendix, item K).

3.2 (In)dependent Propositions

Intuitively, two events are *independent* and have no causal impact on each other if and only if they are evaluated in two unrelated samples. In Sample Logic and in accordance with Probability Theory, we have $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cdot [\![\psi]\!]$ for two independent events; this is an immediate consequence of the definition of conjunction in (19b). However, this equation does not hold if the events are *dependent*. To illustrate this point, consider one pair of independent examples and two pairs of dependent examples, that is, one pair of coordinated and one pair of subordinated examples.

(21) a. ϕ: Clorox will strongly increase sales of its disinfecting products **and** $\quad [\![\phi]\!] = 21\%$
b. ψ: the car rental company Hertz will file for bankruptcy next year. $\quad [\![\psi]\!] = 2\%$
c. $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cdot [\![\psi]\!] < [\![\phi]\!], [\![\psi]\!]$. $\quad [\![\phi \wedge \psi]\!] = 0.5\%$

(22) a. ϕ: Clorox will strongly increase sales of its disinfecting products **and** $\quad [\![\phi]\!] = 21\%$
b. ψ: Kimberley-Clark will greatly boost sales of its paper towels next year. $\quad [\![\psi]\!] = 19\%$
c. $[\![\phi \wedge \psi]\!] < [\![\phi]\!], [\![\psi]\!]$. $\quad [\![\phi \wedge \psi]\!] = 17\%$

(23) a. ϕ: There is a pandemic next year **and** $\quad [\![\phi]\!] = 82\%$
b. ψ: Clorox will strongly increase sales of its disinfecting products. $\quad [\![\psi]\!] = 21\%$
c. $[\![\psi]\!] < [\![\phi \wedge \psi]\!] < [\![\phi]\!]$. $\quad [\![\phi \wedge \psi]\!] = 71\%$

In (21), Clorox and Hertz are mutually independent in their economic performance. In addition to the fact that they do not partner or compete with each other, they also do not share partners or competitors. Their samples are uncorrelated and the conjunctive truth degree is the multiplication of the component degrees (according to 19b).

In (22), by contrast, Clorox and Kimberley-Clark partly depend on each other for their performance. They sell complementary products that final consumers often purchase together. For instance, the online retailer behemoth Amazon claims that purchases of Clorox disinfecting bleach systematically trigger and are triggered by purchases of Scott paper towels, a brand of Kimberley-Clark. The conjunctive truth degree is calculated according to (19c), but does not fall under one of the special cases. (For a detailed breakdown, refer to appendix, item L.)

The economic performance of Clorox in (23) is subordinated to the occurrence of a pandemic. As macroeconomic predictor, epidemics filter out companies in the Clorox sample that did business in a year without epidemic. Of the 349 entries in the Clorox sample, 96 entries relate to companies operating during a past epidemic with the same reproduction (**β**) and recovery (**γ**) rates as Covid-19. About 68 entries refer to a year with a pandemic and to companies that witnessed an increase in their sales by at least nine percent. The majority of comparative data stem from the Spanish Flu in 1918 when the disinfecting industry got started (Stowe 2018). The conjunctive truth degree is determined according to (19d) and is 71% (**P** = 68/96), refer to appendix, item M.

Furthermore, three special cases of coordinated dependency exist, which we label as *interdependency*, *skewed dependency* and *interrelation*. Interdependency refers to two samples with an empty intersection (see 19c-i); skewed dependency denotes a situation where the first sample is strictly included in the second (see 19c-ii); finally, interrelation is a kind of dependency where both samples are identical (19c-iii). Examples (24)-(26) provide an illustration.

(24) a. ϕ: The Dallas Mavericks players are taller than the Utah Jazz… $[\![\phi]\!] = 56\%$

b. ψ: …**and** Denver Nuggets players. $[\![\psi]\!] = 40\%$

c. $[\![\psi]\!] < [\![\phi \wedge \psi]\!] < [\![\phi]\!]$ $[\![\phi \wedge \psi]\!] = 48\%$

(25) a. ϕ: The Dallas Mavericks players are taller than the Utah Jazz players, $[\![\phi]\!] = 56\%$

b. ψ: **but** they are not the tallest team of the NBA. $[\![\psi]\!] = 53\%$

c. $[\![\phi \wedge \psi]\!] < [\![\phi]\!], [\![\psi]\!]$ $[\![\phi \wedge \psi]\!] = 52\%$

(26) a. ϕ: The Dallas Mavericks players are taller **and** $[\![\phi]\!] = 56\%$

b. ψ: heavier than the Utah Jazz players. $[\![\psi]\!] = 54\%$

c. $[\![\phi \wedge \psi]\!] < [\![\phi]\!], [\![\psi]\!]$ $[\![\phi \wedge \psi]\!] = 44\%$

A comparison of the Dallas Mavericks and the Utah Jazz players in (24) is interdependent with the comparison of the Dallas Mavericks and the Denver Nuggets players (refer to appendix, item N). In (25), the comparisons of Dallas Mavericks and Utah Jazz players (289 items) are contained in the much larger comparison class of Dallas Mavericks and NBA players (8160 items). The dependency is skewed towards the second sentence (refer to appendix, item O). In (26), we compare the height and weight of two teams. These variables exhibit an interrelation, the third kind of special dependency (refer to appendix, item P).

For dependent sentences, the conjunctive truth degree is not necessarily smaller than both component degrees. The two interdependent sentences in (24) exemplify the situation where the conjunctive degree (48%) sits in between the component degrees (40% and 56%). The same may also hold true for propositions with skewed dependency. If we make a slight modification in example (25) as in (27), then the relationship of the conjunctive degree and its component degrees tips over (refer to appendix, item Q).

(27) a. ϕ: The Dallas Mavericks players are taller than the Denver Nuggets players, $[\![\phi]\!] = 40\%$

b. ψ: **and** even the tallest team of the NBA. $[\![\psi]\!] = 47\%$

c. $[\![\phi]\!] < [\![\phi \wedge \psi]\!] = [\![\psi]\!]$ $[\![\phi \wedge \psi]\!] = 47\%$

This state of affairs contrasts with general probability functions **P** where we always have $\mathbf{P}(A \cap B) \leq \mathbf{P}(A)$ and $\mathbf{P}(A \cap B) \leq \mathbf{P}(B)$. Yet, the possibility of $[\![\phi]\!] < [\![\phi \wedge \psi]\!]$ is perfectly consistent with language intuition in linguistics. This discrepancy is attributed to the fact that Sample Logic only satisfies the weakened version of Kolmogorov axiom K3. For independent propositions and interrelations, which satisfy axiom K3, we always have $[\![\phi \wedge \psi]\!] \leq [\![\phi]\!], [\![\psi]\!]$. Finally, the conjunction in Sample Logic satisfies the following properties that play a role in evaluating the conditional.

(28) *Lemma (Properties of Conjunction in Sample Logic)*:

a. Conjunctive truth may not surpass **both** component truths: $[\![\phi \wedge \psi]\!] \leq \max([\![\phi]\!], [\![\psi]\!])$.

b. If ϕ and ψ are independent, then $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \Leftrightarrow [\![\phi]\!] = 0 \text{ } or \text{ } [\![\psi]\!] = 1$.

c. If ϕ and ψ are in interrelation, then $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \Rightarrow [\![\phi]\!] \leq [\![\psi]\!]$.

3.3 Conditional and Biconditional

In line with Probability Theory, conditional clauses are interpreted as the truth degree of antecedent *and* consequent divided by the truth degree of the antecedent (Gut 2005: 17; Kaufman 2005: 197). The biconditional *if and only if* ($\phi \leftrightarrow \psi$) cannot be evaluated as the conjunction of the two conditionals $(\phi \to \psi) \wedge (\psi \to \phi)$. This definition is tight to the material conditional used in Classical Logic. In Sample Logic, the biconditional is the quotient of the two component truth degrees depending on which one is greater (and provided that their truth degree is not zero).

(29) *Definition of Conditional and Biconditional:*

Let ϕ and ψ be two formulas with samples $\mathbf{\Sigma}_\phi$, $\mathbf{\Sigma}_\psi$ and property sets $\mathbf{X}_\phi$, $\mathbf{X}_\psi$.

a. The sample and property set of the conditional are defined as follows where $\{\varnothing\}$ is the singleton set:

Property Set: $$\mathbf{X}_{\phi\to\psi} = \begin{cases} \mathbf{X}_{\phi\wedge\psi} \times \mathbf{\Sigma}_\phi \ \textit{if}\ \mathbf{X}_\phi \neq \varnothing \ \textit{and}\ \left|\mathbf{X}_{\phi\wedge\psi} \times \mathbf{\Sigma}_\phi\right| < \left|\mathbf{\Sigma}_{\phi\wedge\psi} \times \mathbf{X}_\phi\right| \\ \mathbf{\Sigma}_{\phi\wedge\psi} \times \mathbf{X}_\phi \ \textit{if}\ \mathbf{X}_\phi \neq \varnothing \ \textit{and}\ \left|\mathbf{\Sigma}_{\phi\wedge\psi} \times \mathbf{X}_\phi\right| < \left|\mathbf{X}_{\phi\wedge\psi} \times \mathbf{\Sigma}_\phi\right| \\ \{\varnothing\} \ \textit{if}\ \mathbf{X}_\phi = \varnothing \ \textit{or}\ \left|\mathbf{X}_{\phi\wedge\psi} \times \mathbf{\Sigma}_\phi\right| = \left|\mathbf{\Sigma}_{\phi\wedge\psi} \times \mathbf{X}_\phi\right| \end{cases}$$

Sample: $$\mathbf{\Sigma}_{\phi\to\psi} = \begin{cases} \mathbf{\Sigma}_{\phi\wedge\psi} \times \mathbf{X}_\phi \ \textit{if}\ \mathbf{X}_\phi \neq \varnothing \ \textit{and}\ \left|\mathbf{X}_{\phi\wedge\psi} \times \mathbf{\Sigma}_\phi\right| < \left|\mathbf{\Sigma}_{\phi\wedge\psi} \times \mathbf{X}_\phi\right| \\ \mathbf{X}_{\phi\wedge\psi} \times \mathbf{\Sigma}_\phi \ \textit{if}\ \mathbf{X}_\phi \neq \varnothing \ \textit{and}\ \left|\mathbf{\Sigma}_{\phi\wedge\psi} \times \mathbf{X}_\phi\right| < \left|\mathbf{X}_{\phi\wedge\psi} \times \mathbf{\Sigma}_\phi\right| \\ \{\varnothing\} \ \textit{if}\ \mathbf{X}_\phi = \varnothing \ \textit{or}\ \left|\mathbf{X}_{\phi\wedge\psi} \times \mathbf{\Sigma}_\phi\right| = \left|\mathbf{\Sigma}_{\phi\wedge\psi} \times \mathbf{X}_\phi\right| \end{cases}$$

b. The biconditional is interpreted by the following sample and property set:

Property Set: $$\mathbf{X}_{\phi\leftrightarrow\psi} = \begin{cases} \mathbf{X}_\psi \times \mathbf{\Sigma}_\phi \ \textit{if}\ \mathbf{X}_\phi \neq \varnothing, \mathbf{X}_\psi \neq \varnothing \ \textit{and}\ \left|\mathbf{X}_\psi \times \mathbf{\Sigma}_\phi\right| < \left|\mathbf{X}_\phi \times \mathbf{\Sigma}_\psi\right| \\ \mathbf{X}_\phi \times \mathbf{\Sigma}_\psi \ \textit{if}\ \mathbf{X}_\phi \neq \varnothing, \mathbf{X}_\psi \neq \varnothing \ \textit{and}\ \left|\mathbf{X}_\phi \times \mathbf{\Sigma}_\psi\right| < \left|\mathbf{X}_\psi \times \mathbf{\Sigma}_\phi\right| \\ \varnothing \ \textit{if}\ \left(\mathbf{X}_\phi = \varnothing \ \textit{and}\ \mathbf{X}_\psi \neq \varnothing\right) \ \textit{or}\ \left(\mathbf{X}_\phi \neq \varnothing \ \textit{and}\ \mathbf{X}_\psi = \varnothing\right) \\ \{\varnothing\} \ \textit{if}\ \left|\mathbf{X}_\phi \times \mathbf{\Sigma}_\psi\right| = \left|\mathbf{X}_\psi \times \mathbf{\Sigma}_\phi\right| \end{cases}$$

Sample: $$\mathbf{\Sigma}_{\phi\leftrightarrow\psi} = \begin{cases} \mathbf{X}_\phi \times \mathbf{\Sigma}_\psi \ \textit{if}\ \mathbf{X}_\phi \neq \varnothing, \mathbf{X}_\psi \neq \varnothing \ \textit{and}\ \left|\mathbf{X}_\psi \times \mathbf{\Sigma}_\phi\right| < \left|\mathbf{X}_\phi \times \mathbf{\Sigma}_\psi\right| \\ \mathbf{X}_\psi \times \mathbf{\Sigma}_\phi \ \textit{if}\ \mathbf{X}_\phi \neq \varnothing, \mathbf{X}_\psi \neq \varnothing \ \textit{and}\ \left|\mathbf{X}_\phi \times \mathbf{\Sigma}_\psi\right| < \left|\mathbf{X}_\psi \times \mathbf{\Sigma}_\phi\right| \\ \{\varnothing\} \ \textit{if}\ \left|\mathbf{X}_\phi \times \mathbf{\Sigma}_\psi\right| = \left|\mathbf{X}_\psi \times \mathbf{\Sigma}_\phi\right| \ \textit{or}\ \left(\mathbf{X}_\phi = \varnothing \ \textit{or}\ \mathbf{X}_\psi = \varnothing\right) \end{cases}$$

c. The truth degrees of the conditional and biconditional are defined as follows:

$[\![\phi \to \psi]\!] = \dfrac{\left|\mathbf{X}_{\phi\to\psi}\right|}{\left|\mathbf{\Sigma}_{\phi\to\psi}\right|}$ and $[\![\phi \leftrightarrow \psi]\!] = \dfrac{\left|\mathbf{X}_{\phi\leftrightarrow\psi}\right|}{\left|\mathbf{\Sigma}_{\phi\leftrightarrow\psi}\right|}$.

These bulky samples and property sets do ensure that the conditional and biconditional are probabilistic, but the conditional and biconditionals need to flip both the numerator and denominator when the conjunctive truth degrees surpasses one of the component truth degree. The following lemma summarizes the relationship shared between the conjunction, conditional, and biconditional (for proof, refer to appendix, item R).

(30) *Lemma*:

a. $$\llbracket \phi \rightarrow \psi \rrbracket = \begin{cases} \dfrac{\llbracket \phi \wedge \psi \rrbracket}{\llbracket \phi \rrbracket} & if\ \llbracket \phi \rrbracket > 0\ and\ \llbracket \phi \wedge \psi \rrbracket < \llbracket \phi \rrbracket \\ \dfrac{\llbracket \phi \rrbracket}{\llbracket \phi \wedge \psi \rrbracket} & if\ \llbracket \phi \rrbracket > 0\ and\ \llbracket \phi \rrbracket < \llbracket \phi \wedge \psi \rrbracket \end{cases}$$

b. $\llbracket \phi \rightarrow \psi \rrbracket = 0$ iff $\llbracket \phi \rrbracket > 0$ *and* $\llbracket \phi \wedge \psi \rrbracket = 0$

c. $\llbracket \phi \rightarrow \psi \rrbracket = 1$ iff $\llbracket \phi \rrbracket = 0$ *or* $\llbracket \phi \wedge \psi \rrbracket = \llbracket \phi \rrbracket$

d. $$\llbracket \phi \leftrightarrow \psi \rrbracket = \begin{cases} \dfrac{\llbracket \phi \rightarrow \psi \rrbracket}{\llbracket \psi \rightarrow \phi \rrbracket} = \dfrac{\llbracket \psi \rrbracket}{\llbracket \phi \rrbracket} & if\ \llbracket \phi \rrbracket > 0, \llbracket \psi \rrbracket > 0\ and\ 0 < \llbracket \phi \wedge \psi \rrbracket \le \llbracket \psi \rrbracket \le \llbracket \phi \rrbracket \\ \dfrac{\llbracket \psi \rightarrow \phi \rrbracket}{\llbracket \phi \rightarrow \psi \rrbracket} = \dfrac{\llbracket \phi \rrbracket}{\llbracket \psi \rrbracket} & if\ \llbracket \phi \rrbracket > 0, \llbracket \psi \rrbracket > 0\ and\ 0 < \llbracket \phi \wedge \psi \rrbracket \le \llbracket \phi \rrbracket < \llbracket \psi \rrbracket \\ \llbracket \phi \rightarrow \psi \rrbracket \cdot \llbracket \psi \rightarrow \phi \rrbracket = \dfrac{\llbracket \psi \rrbracket}{\llbracket \phi \rrbracket} & if\ 0 < \llbracket \psi \rrbracket < \llbracket \phi \wedge \psi \rrbracket \le \llbracket \phi \rrbracket \\ \llbracket \phi \rightarrow \psi \rrbracket \cdot \llbracket \psi \rightarrow \phi \rrbracket = \dfrac{\llbracket \phi \rrbracket}{\llbracket \psi \rrbracket} & if\ 0 < \llbracket \phi \rrbracket < \llbracket \phi \wedge \psi \rrbracket \le \llbracket \psi \rrbracket \end{cases}$$

e. $\llbracket \phi \leftrightarrow \psi \rrbracket = 0$ iff ($\llbracket \phi \rrbracket = 0$ and $\llbracket \psi \rrbracket > 0$) *or* ($\llbracket \phi \rrbracket > 0$ and $\llbracket \psi \rrbracket = 0$)

f. $\llbracket \phi \leftrightarrow \psi \rrbracket = 1$ iff $\llbracket \phi \rrbracket = \llbracket \psi \rrbracket$

g. If ϕ and ψ are independent, then $\llbracket \phi \rightarrow \psi \rrbracket = 1 \Leftrightarrow \llbracket \phi \rrbracket = 0\ or\ \llbracket \psi \rrbracket = 1$

h. If ϕ and ψ are in interrelation, then $\llbracket \phi \rightarrow \psi \rrbracket = 1 \Rightarrow \llbracket \phi \rrbracket \le \llbracket \psi \rrbracket$.

The probabilistic conditional is reflective of basic language intuition of *if*-clauses in human language, and the biconditional of *if and only if*-clauses. When two independent propositions occur as the antecedent and consequent, as in (31), then the truth degree of the conditional is equal to the truth degree of the consequent. The following examples denote modifications of the conjunctions (21)-(27). For the calculation of the truth degree, refer to appendix, item S.

(31) a. ϕ: (**If**) Clorox will strongly increase sales of its disinfecting products (/**iff**)
b. ψ: (**then**) the car rental company Hertz will file for bankruptcy next year.

$$\left\langle \begin{matrix} \llbracket \phi \rrbracket = 21\% \\ \llbracket \psi \rrbracket = 2\% \end{matrix} \middle| \begin{matrix} \llbracket \phi \rightarrow \psi \rrbracket = 2\% \\ \llbracket \psi \rightarrow \phi \rrbracket = 21\% \\ \llbracket \phi \leftrightarrow \psi \rrbracket = 11\% \end{matrix} \right\rangle$$

(32) a. ϕ: (**If**) Clorox will strongly increase sales of its disinfecting products (/**iff**)
b. ψ: (**then**) Kimberley-Clark will greatly boost sales of its paper towels next year.

$$\left\langle \begin{matrix} \llbracket \phi \rrbracket = 21\% \\ \llbracket \psi \rrbracket = 19\% \end{matrix} \middle| \begin{matrix} \llbracket \phi \rightarrow \psi \rrbracket = 84\% \\ \llbracket \psi \rightarrow \phi \rrbracket = 93\% \\ \llbracket \phi \leftrightarrow \psi \rrbracket = 91\% \end{matrix} \right\rangle$$

(33) a. ϕ: (**If**) there is a pandemic next year (/**iff**)
b. ψ: (**then**) Clorox will strongly increase sales of its disinfecting products.

$$\left\langle \begin{matrix} \llbracket \phi \rrbracket = 82\% \\ \llbracket \psi \rrbracket = 21\% \end{matrix} \middle| \begin{matrix} \llbracket \phi \rightarrow \psi \rrbracket = 87\% \\ \llbracket \psi \rightarrow \phi \rrbracket = 30\% \\ \llbracket \phi \leftrightarrow \psi \rrbracket = 26\% \end{matrix} \right\rangle$$

(34) a. ϕ: (**If**) the Dallas Mavericks players are taller than the Utah Jazz players (/**iff**)
b. ψ: (**then**) they are also taller than the Denver Nuggets players.

$$\left\langle \begin{matrix} \llbracket \phi \rrbracket = 56\% \\ \llbracket \psi \rrbracket = 40\% \end{matrix} \middle| \begin{matrix} \llbracket \phi \rightarrow \psi \rrbracket = 86\% \\ \llbracket \psi \rightarrow \phi \rrbracket = 83\% \\ \llbracket \phi \leftrightarrow \psi \rrbracket = 71\% \end{matrix} \right\rangle$$

(35) a. ϕ: (**If**) the Dallas Mavericks players are taller than the Utah Jazz players (/**iff**)
b. ψ: (**then**) they are also heavier than the Utah Jazz players.

$$\left\langle \begin{matrix} \llbracket \phi \rrbracket = 56\% \\ \llbracket \psi \rrbracket = 54\% \end{matrix} \middle| \begin{matrix} \llbracket \phi \rightarrow \psi \rrbracket = 79\% \\ \llbracket \psi \rightarrow \phi \rrbracket = 82\% \\ \llbracket \phi \leftrightarrow \psi \rrbracket = 96\% \end{matrix} \right\rangle$$

(36) a. ϕ: (**If**) the Dallas Mavericks players are taller than the Denver Nuggets players (/**iff**)
b. ψ: (**then**) they are even the tallest team of the NBA.

$$\left\langle \begin{matrix} [\![\phi]\!] = 40\% \\ [\![\psi]\!] = 47\% \end{matrix} \middle| \begin{matrix} [\![\phi \rightarrow \psi]\!] = 74\% \\ [\![\psi \rightarrow \phi]\!] = 100\% \\ [\![\phi \leftrightarrow \psi]\!] = 74\% \end{matrix} \right\rangle$$

In Sample Logic, the conditional is not a (fuzzy) residual conditional, because, when $[\![\phi]\!] \leq [\![\psi]\!]$, it is not necessarily the case that $[\![\phi \rightarrow \psi]\!] = 1$. (36) provides a counterexample. Furthermore, neither is the conditional in Sample Logic a material conditional, nor is the biconditional a material biconditional. In fact, the truth degrees of both the material conditional and biconditional are far apart from their probabilistic counterparts, as evidenced in examples (31)-(36). For the calculation of the material truth degrees, refer to appendix, item T.

Example	$[\![\phi]\!]$	$[\![\psi]\!]$	$[\![\phi \rightarrow \psi]\!]$	$[\![\neg\phi \vee \psi]\!]$	$[\![\phi \leftrightarrow \psi]\!]$	$[\![(\neg\phi \vee \psi) \wedge (\neg\psi \vee \phi)]\!]$
(31)	21%	2%	2%	$80\% = \frac{59981}{75384}$	11%	$78\% = \frac{58601}{75384}$
(32)	21%	19%	84%	$56\% = \frac{350}{627}$	91%	$20\% = \frac{128}{627}$
(33)	82%	21%	86%	$98\% = \frac{94}{96}$	26%	$94\% = \frac{90}{96}$
(34)	56%	40%	86%	$42\% = \frac{235}{561}$	71%	0%
(35)	56%	54%	79%	$88\% = \frac{255}{289}$	96%	$83\% = \frac{241}{289}$
(36)	40%	47%	74%	$48\% = \frac{3920}{8160}$	74%	$3\% = \frac{272}{8160}$

Table 2: Comparison of the Material and Probabilistic Conditionals/Biconditionals

The truth degrees of the material conditionals seem counterintuitive. For example, the truth degree of (32) (***If*** *Clorox strongly increase sales of its disinfecting products next year,* ***then*** *Kimberley-Clark will greatly boost sales of its paper towels*), which is 56%, appears to be too low. Since both companies are correlated, good performance of one is likely to entail good performance of the other. By contrast, the corresponding truth degree of the probabilistic conditional (which is 84%) appears to be more reasonable.

However, if we consider $\neg\phi \vee \psi$ as inequivalent to the conditional and interpret it as a disjunction of two *related alternatives* (Simons 2001), then the truth degrees reflect basic language intuition: *Clorox will* ***not*** *strongly increase sales of its disinfecting products next year,* ***or*** *Kimberley-Clark will greatly boost sales of its paper towels*. The truth degree of 56% indicates that the two alternatives occupy approximately half of all possibilities (other options include strong performance by Clorox and mediocre to bad performance by Kimberley-Clark at about 44%).

3.4 Conditional (In)dependence

In the case of complex conditionals, the antecedent and/or the consequent are complex sentences. I shall concentrate on conditionals where the consequent is a conjunction and, in particular, on the notion of *conditional independence*. (Unconditional) independence is defined by the conjunctive property $[\![\phi \wedge \psi]\!] = [\![\phi]\!] \cdot [\![\psi]\!]$ discussed in section 3.2. The most common formal definition of conditional independence is as follows (Spohn 1980, Studený 2002):[32]

(37) *Definition (Conditional Independence)*

ϕ and ψ are conditionally independent of χ iff $[\![\chi \rightarrow (\phi \wedge \psi)]\!] = [\![\chi \rightarrow \phi]\!] \cdot [\![\chi \rightarrow \psi]\!]$.

[32] Humberstone (2020) discusses a range of alternative (proof-theoretic) concepts of independency.

We can understand *unconditional dependence* as a proxy for the concept of *direct causation* where the occurrence of an event causes the manifestation of another dependent event. *Conditional dependence* then represents the idea of *indirect causation*.[33] Two dependent events may be falsely viewed as causes of one another until a third event becomes known that is the real cause for one or both events. Spohn (1980) presents several entertaining examples of correlation studies in social sciences.

As two distinct properties, unconditional (in)dependence (of two events ϕ and ψ) and conditional independence (of ϕ and ψ relative to χ) do not entail each other. In Sample Logic, the following relationship between conditional and unconditional independence holds (for proof, refer to appendix, item U).[34]

(38) *Lemma*

a. If $\left\{\begin{array}{l}\text{(i) } \phi \text{ and } \psi \text{ are independent, and} \\ \text{(ii) } (\chi \text{ and } \phi) \ \underline{\textbf{or}} \ (\chi \text{ and } \psi) \text{ are independent,}\end{array}\right\}$ then $\left\{\begin{array}{l}\phi \text{ and } \psi \text{ are conditionally} \\ \text{independent of } \chi.\end{array}\right.$

b. If $\left\{\begin{array}{l}\text{(i) } \phi \text{ and } \psi \text{ are conditionally independent of } \chi \text{ and} \\ \text{(ii) } (\chi \text{ and } \phi) \ \underline{\textbf{and}} \ (\chi \text{ and } \psi) \text{ are independent,}\end{array}\right\}$ then $\{\phi \text{ and } \psi \text{ are independent.}$

This result is in close alignment with linguistic intuition that I shall illustrate with three examples. In (39), the antecedent and consequent are both conditionally and unconditionally independent. Meanwhile, in (40), they are unconditionally independent and conditionally dependent. Similarly, in (41), they are both conditionally and unconditionally dependent (for the calculation of the truth degrees, refer to appendix, item V).

(39) *ϕ and ψ are mutually independent and conditionally independent of χ*

a. χ: **If** there is a recession next year, **then**
b. ϕ: Clorox will strongly increase sales of its disinfecting products **and**
c. ψ: Bimbo will neither hike nor diminish its bakery business.
d. $[\![\chi \to (\phi \wedge \psi)]\!] = 9\%$ and $[\![\chi \to \phi]\!] \cdot [\![\chi \to \psi]\!] = 9\%$.

$$\left\langle \begin{array}{l|l} [\![\chi]\!] = 64\% & [\![\phi \wedge \psi]\!] = 9\% \\ [\![\phi]\!] = 21\% & [\![\phi]\!] \cdot [\![\psi]\!] = 9\% \\ [\![\psi]\!] = 41\% & [\![\chi \to \phi]\!] = 21\% \\ & [\![\chi \to \psi]\!] = 41\% \end{array} \right\rangle$$

(40) *ϕ and ψ are mutually independent and conditionally dependent of χ*

a. χ: **If** there is a pandemic next year, **then**
b. ϕ: Clorox will strongly increase sales of its disinfecting products **and**
c. ψ: the car rental company Hertz will file for bankruptcy.
d. $[\![\chi \to (\phi \wedge \psi)]\!] = 95\%$ and $[\![\chi \to \phi]\!] \cdot [\![\chi \to \psi]\!] = 62\%$.

$$\left\langle \begin{array}{l|l} [\![\chi]\!] = 82\% & [\![\phi \wedge \psi]\!] = 0.5\% \\ [\![\phi]\!] = 21\% & [\![\phi]\!] \cdot [\![\psi]\!] = 0.5\% \\ [\![\psi]\!] = 2\% & [\![\chi \to \phi]\!] = 87\% \\ & [\![\chi \to \psi]\!] = 72\% \end{array} \right\rangle$$

(41) *ϕ and ψ are mutually dependent and conditionally dependent of χ*

a. χ: **If** there is a pandemic next year, **then**
b. ϕ: Clorox will strongly increase sales of its disinfecting products **and**
c. ψ: Kimberley-Clark will do the same for its Scott paper towels.
d. $[\![\chi \to (\phi \wedge \psi)]\!] = 91\%$ and $[\![\chi \to \phi]\!] \cdot [\![\chi \to \psi]\!] = 59\%$.

$$\left\langle \begin{array}{l|l} [\![\chi]\!] = 82\% & [\![\phi \wedge \psi]\!] = 17\% \\ [\![\phi]\!] = 21\% & [\![\phi]\!] \cdot [\![\psi]\!] = 4\% \\ [\![\psi]\!] = 19\% & [\![\chi \to \phi]\!] = 87\% \\ & [\![\chi \to \psi]\!] = 68\% \end{array} \right\rangle$$

[33] For the broad linguistic literature on causation, refer to Cruse (1972), Fodor (1970) and Wolff (2003).

[34] It is not possible to prove Lemma (38) in general Probabilistic Logic, because we do not know whether from the independence of α and γ and from the independence of β and γ we can conclude that $(\alpha \wedge \beta)$ and γ are independent.

In (39), Clorox and Bimbo represent two global firms that are mutually independent in their performance. Both are impervious to economic recessions because the products they manufacture are needed both in good times and bad times.

By contrast, the car rental company Hertz in (40) is frail to external shocks like recessions or pandemics. The business operations of Clorox are also affected during a pandemic, which serves as a catalyst, and not as an impediment. Although Hertz and Clorox operate autonomously, their businesses are codependent on a pandemic if one occurs. This situation is reflected by the truth degrees.

Kimberley-Clark, a global manufacturer of consumer tissues, is susceptible to pandemics in a similar way to that of Clorox. The business operations of both firms are mutually dependent under normal circumstances. In the rate occurrence of a pandemic, they cling even more together, as indicated by the truth degrees in (41).

4. Conclusion

Sample Logic belongs to the family of probabilistic logics, allows a constructive compounding of truth degrees, obviates challenges related to coherence, and satisfies a range of expected properties (such as complementary negation or the de Morgan laws). Most importantly, Sample Logic bakes different types of sentence dependency into its semantics that do justice to linguistic data. The selected (bi)conditional in Sample Logic is the probabilistic one that is more natural than the material and residual (bi)conditionals. The concept of conditional dependency, which serves as a proxy of the linguistic idea of indirect causation, is another significant asset of Sample Logic.

Over the past 50 years, linguists and philosophers introduced ontologies for individuals, events, situations, possible worlds, and degrees on top of the Classical Bivalent Logic in order to handle different sets of linguistic data. This situation has led to a proliferation of mutually incompatible logics. In the event a piece of linguistic data makes simultaneously reference to, say, degrees and possible worlds, then the “standard” linguistic analysis might quickly get opaque and crowded. Consider an example.

(42) a. ϕ: **If** it is hot next week,
b. ψ: **then** John might go to the swimming-pool.

$$\left\langle \begin{array}{c|c} [\![\phi]\!] = 50\% & [\![\phi \wedge \psi]\!] = 40\% \\ [\![\psi]\!] = 60\% & [\![\phi \rightarrow \psi]\!] = 80\% \end{array} \right\rangle$$

Hitherto, the evaluation of (42) would integrate the notions of degree and possible worlds into the truth conditions, thus resulting in complex expressions. In Sample Logic, the conjunctive truth degree reveals precise information about the extent to which John’s recreation plans are contingent on hot weather conditions. The truth degree of the probabilistic conditional provides a semantic evaluation of (42).

Appendix

Item A (Section 1.2)

Sauerland renewed Kamp’s argument against fuzzy logic uses the *Fixed-Point Theorem* of the Dutch mathematician Luitzen Brouwer (1881-1966) but has a gap.

(43) *Sauerland (2011)’s argument against Fuzzy Logic*:

Sauerland assumes that any fuzzy negation that is supposed to be useful in linguistics must be a continuous function $\neg$: [0,1] $\rightarrow$ [0,1], not a function that jumps. If the negation $\neg$ is continuous, then Brouwer’s well-known *Fixed-Point Theorem* (“Every continuous function f: [0,1] $\rightarrow$ [0,1] has a fixed point: $x_0 \in [0,1]$ with $f(x_0) = x_0$”) guarantees that there is value $x_0 \in [0,1]$ with $\neg x_0 = x_0$. This value may or may not be instantiated by a sentence ϕ (and here is the gap in his argumentation). If it is instantiated, then we have $\llbracket\neg\phi\rrbracket = \llbracket\phi\rrbracket$. From the decreasing properties of *t-norms* it follows that $\llbracket\neg\phi \wedge \neg\phi\rrbracket = \llbracket\phi \wedge \neg\phi\rrbracket = \llbracket\phi \wedge \phi\rrbracket$. With this being the case, it is not possible that $\llbracket\cdot\rrbracket$ is both coherent and idempotent, because, on the one hand, the truth degree of the contradiction $\phi \wedge \neg\phi$ must be zero. On the other hand, $\llbracket\phi \wedge \phi\rrbracket$ and $\llbracket\neg\phi \wedge \neg\phi\rrbracket$ cannot be both zero. Hence there is a problem with fuzzy logic. In order to make a valid point against Fuzzy Logic, Sauerland would need to prove that whatever the evaluation of atomic proposition $\llbracket\cdot\rrbracket$ and whatever the continuous fuzzy negation $\neg$ are, they cannot simultaneously satisfy coherence and idempotency. Since fuzzy evaluations are not required to be surjective functions, we do not know whether the fix point of a particular fuzzy negation is instantiated by a sentence ϕ. If it is not instantiated, it would not follow that $\llbracket\cdot\rrbracket$ is either incoherent or non-idempotent.

Below is my proof of the lemma in section 1.2, a renewal of Kamp's argument against fuzzy logic which makes full use of t-norms. As soon as we require the negation not to be reduced to zero and one (it does not even need to be a continuous function), Kamp's argument follows suit.

(44) Lemma

a. Let $\otimes : [0,1]^2 \to [0,1]$ be a t-norm on which a fuzzy evaluation $\llbracket\phi \wedge \psi\rrbracket = \llbracket\phi\rrbracket \otimes \llbracket\psi\rrbracket$ is based. **If** the evaluation $\llbracket\cdot\rrbracket$ simultaneously satisfy the properties of *idempotency* ($\llbracket\phi \wedge \phi\rrbracket = \llbracket\phi\rrbracket$) and of *coherence* ($\llbracket\phi \wedge \neg\phi\rrbracket = 0$), **then** the negation is reduced to 0 and 1 (for all ϕ: $\llbracket\neg\phi\rrbracket = 0$ or 1).

b. *Proof*: Suppose that $\llbracket\cdot\rrbracket$ satisfies the properties of *idempotency* and *coherence*. Consider the case where $0 < \llbracket\phi\rrbracket \leq \llbracket\neg\phi\rrbracket$. As the t-norm $\otimes$ is a monotone non-decreasing function in both arguments, we have $\llbracket\phi\rrbracket \otimes \llbracket\phi\rrbracket \leq \llbracket\phi\rrbracket \otimes \llbracket\neg\phi\rrbracket$. As $\llbracket\cdot\rrbracket$ is idempotent, we have $0 < \llbracket\phi\rrbracket \otimes \llbracket\phi\rrbracket \leq \llbracket\phi\rrbracket \otimes \llbracket\neg\phi\rrbracket$. It follows that $\llbracket\cdot\rrbracket$ is not coherent. From this contradiction we conclude that one of the other two cases must hold, either ($\llbracket\phi\rrbracket = 0$ and $\llbracket\neg\phi\rrbracket = 1$) or ($\llbracket\neg\phi\rrbracket < \llbracket\phi\rrbracket$). We only need to pursue the last case. Again, as the t-norm $\otimes$ is a monotone non-decreasing function in both arguments, it follows that $\llbracket\neg\phi\rrbracket \otimes \llbracket\neg\phi\rrbracket \leq \llbracket\neg\phi\rrbracket \otimes \llbracket\phi\rrbracket \leq \llbracket\phi\rrbracket \otimes \llbracket\phi\rrbracket$. As $\llbracket\cdot\rrbracket$ is idempotent, we can reduce the inequality to $\llbracket\neg\phi\rrbracket \leq \llbracket\neg\phi\rrbracket \otimes \llbracket\phi\rrbracket \leq \llbracket\phi\rrbracket$. As $\llbracket\cdot\rrbracket$ is coherent, we have $0 = \llbracket\neg\phi\rrbracket = \llbracket\neg\phi\rrbracket \otimes \llbracket\phi\rrbracket$ what we aimed to show.

Item B (Section 1.4)

The semantics of Classical Bivalent Logic and Kleene Trivalent Logic (Priest 2001: 122) are defined by truth tables (where 0.5 can be interpreted as "half true" or "neither true nor false").

(45) Classical Bivalent Logic

ϕ	$\neg\phi$
1	0
0	1

$\phi \wedge \psi$	1	0
1	1	0
0	0	0

$\phi \vee \psi$	1	0
1	1	1
0	1	0

$\neg\phi \vee \psi$	1	0
1	1	0
0	1	1

$\phi \to \psi$	1	0
1	1	0
0	1	1

(46) Kleene Trivalent Logic

ϕ	$\neg\phi$
1	0
0.5	0.5
0	1

$\phi \wedge \psi$	1	0.5	0
1	1	0.5	0
0.5	0.5	0.5	0
0	0	0	0

$\phi \vee \psi$	1	0.5	0
1	1	1	1
0.5	1	0.5	0.5
0	1	0.5	0

$\neg\phi \vee \psi$	1	0.5	0
1	1	0.5	0
0.5	1	0.5	0.5
0	1	1	1

$\phi \to \psi$	1	0.5	0
1	1	0.5	0
0.5	1	0.5	0.5
0	1	1	1

The truth degrees of the more complex formulas of Table 1 (section 1.4) are computed in Classical Bivalent Logic and Kleene Trivalent Logic as follows. The red color shows non-equivalent values.

(47) Classical Bivalent Logic

$[\![\phi]\!]$	$[\![\psi]\!]$	$[\![\neg\phi\vee\psi]\!]$	$\frac{[\![\phi\wedge\psi]\!]}{[\![\phi]\!]}$ *if* $[\![\phi]\!]>0$	$[\![(\neg\phi\vee\psi)\wedge(\phi\vee\neg\psi)]\!]$	$\begin{cases}[\![\psi]\!]/[\![\phi]\!]\ if\ 0<[\![\psi]\!]\leq[\![\phi]\!]\\ [\![\phi]\!]/[\![\psi]\!]\ if\ 0<[\![\phi]\!]\leq[\![\psi]\!]\end{cases}$
1	1	1	1	1	1
1	0	0	0	0	---
0	1	1	---	0	---
0	0	1	---	1	---

(48)

$[\![\chi]\!]$	$[\![\phi]\!]$	$[\![\psi]\!]$	$[\![\chi\to(\phi\wedge\psi)]\!]$	$[\![\chi\to\phi]\!]\cdot[\![\chi\to\psi]\!]$
1	1	1	1	1
1	1	0	0	0
1	0	1	0	1
1	0	0	0	0
0	1	1	1	1
0	1	0	1	1
0	0	1	1	1
0	0	0	1	1

(47) shows that in Classical Bivalent Logic the material conditional is also a probabilistic and residual conditional. The row in red of (48) demonstrates that the concept of conditional (in)dependence cannot be defined in general in Classical Bivalent Logic.

(49) Kleene Trivalent Logic

$[\![\phi]\!]$	$[\![\psi]\!]$	$[\![\neg\phi\vee\psi]\!]$	$\frac{[\![\phi\wedge\psi]\!]}{[\![\phi]\!]}$ *if* $[\![\phi]\!]>0$	$[\![(\neg\phi\vee\psi)\wedge(\phi\vee\neg\psi)]\!]$	$\begin{cases}[\![\psi]\!]/[\![\phi]\!]\ if\ 0<[\![\psi]\!]\leq[\![\phi]\!]\\ [\![\phi]\!]/[\![\psi]\!]\ if\ 0<[\![\phi]\!]\leq[\![\psi]\!]\end{cases}$
1	1	1	1	1	1
1	0.5	0.5	0.5	0.5	0.5
1	0	0	0	0	---
0.5	1	1	1	0.5	0.5
0.5	0.5	0.5	1	0.5	1
0.5	0	0.5	0	0.5	---
0	1	1	---	0	---
0	0.5	1	---	0.5	---
0	0	1	---	1	---

(50)

$[\![\chi]\!]$	$[\![\phi]\!]$	$[\![\psi]\!]$	$[\![\chi\to(\phi\wedge\psi)]\!]$	$[\![\chi\to\phi]\!]\cdot[\![\chi\to\psi]\!]$
1	1	1	1	1
1	1	0.5	0.5	0.5
1	1	0	0	0
1	0.5	1	0.5	0.5
1	0.5	0.5	0.5	0.25
1	0.5	0	0	0
1	0	1	0	0
1	0	0.5	0	0
1	0	0	0	0
0.5	1	1	1	1
0.5	1	0.5	0.5	0.5
0.5	1	0	0.5	0.5
0.5	0.5	1	0.5	0.5
0.5	0.5	0.5	0.5	0.25
0.5	0.5	0	0.5	0.25
0.5	0	1	0.5	0.5
0.5	0	0.5	0.5	0.25
0.5	0	0	0.5	0.25

$[\![\chi]\!]$	$[\![\phi]\!]$	$[\![\psi]\!]$	$[\![\chi\to(\phi\wedge\psi)]\!]$	$[\![\chi\to\phi]\!]\cdot[\![\chi\to\psi]\!]$
0	1	1	1	1
0	1	0.5	1	1
0	1	0	1	1
0	0.5	1	1	1
0	0.5	0.5	1	1
0	0.5	0	1	1
0	0	1	1	1
0	0	0.5	1	1
0	0	0	1	1

(49) proves that the material conditional in Kleene Trivalent Logic is neither a residual nor a probabilistic conditional. The rows in red font in (50) shows that the concept of conditional (in)dependency has no foundation in this logic.

In Fuzzy Logic, the ‘residual conditional’ generalizes this property of the material conditional in Classical Bivalent Logic: $\llbracket \phi \rightarrow \psi \rrbracket = 1 \Leftrightarrow \llbracket \phi \rrbracket \leq \llbracket \psi \rrbracket$. It is defined as $x \rightarrow y = \max\{z \mid x{*}z \leq y\}$ ($*$ is a t-norm). Fuzzy negation is derived from the conditional by $\neg x = x \rightarrow 0$, see Hájek (1998: 28-32) and Grabowski (2017: 93-95). The ‘residual biconditional’ $\phi \leftrightarrow \psi$ is defined as conjunction of the two residual conditionals $(\phi \rightarrow \psi) \wedge (\psi \rightarrow \phi)$, see Hájek (1998: 66). The three most common fuzzy logics are the Łukasiewicz, Gödel and Product Logics, abbreviated as Ł, L_{Min} and L_{Prod}.

(51)	Fuzzy Logics	*Łukasiewicz Logic*	*Gödel Logic*	*Product Logic*
a.	$\neg\phi$	$1-\llbracket \phi \rrbracket$	$\begin{cases} 1 \text{ if } \llbracket \phi \rrbracket = 0 \\ 0 \text{ if } \llbracket \phi \rrbracket > 0 \end{cases}$	$\begin{cases} 1 \text{ if } \llbracket \phi \rrbracket = 0 \\ 0 \text{ if } \llbracket \phi \rrbracket > 0 \end{cases}$
b.	$\phi \wedge \psi$	$\max\{0, \llbracket \phi \rrbracket + \llbracket \psi \rrbracket - 1\}$	$\min\{\llbracket \phi \rrbracket, \llbracket \psi \rrbracket\}$	$\llbracket \phi \rrbracket \cdot \llbracket \psi \rrbracket$
c.	$\phi \vee \psi$	$\min\{\llbracket \phi \rrbracket + \llbracket \psi \rrbracket, 1\}$	$\max\{\llbracket \phi \rrbracket, \llbracket \psi \rrbracket\}$	$\llbracket \phi \rrbracket + \llbracket \psi \rrbracket - (\llbracket \phi \rrbracket \cdot \llbracket \psi \rrbracket)$
d.	$\phi \rightarrow \psi$	$\begin{cases} 1 \text{ if } \llbracket \phi \rrbracket \leq \llbracket \psi \rrbracket \\ 1 - \llbracket \phi \rrbracket + \llbracket \psi \rrbracket \text{ if } \llbracket \phi \rrbracket > \llbracket \psi \rrbracket \end{cases}$	$\begin{cases} 1 \text{ if } \llbracket \phi \rrbracket \leq \llbracket \psi \rrbracket \\ \llbracket \psi \rrbracket \text{ if } \llbracket \phi \rrbracket > \llbracket \psi \rrbracket \end{cases}$	$\begin{cases} 1 \text{ if } \llbracket \phi \rrbracket \leq \llbracket \psi \rrbracket \\ \frac{\llbracket \psi \rrbracket}{\llbracket \phi \rrbracket} \text{ if } \llbracket \phi \rrbracket > \llbracket \psi \rrbracket \end{cases}$
e.	$\phi \leftrightarrow \psi$	$\begin{cases} 1 \text{ if } \llbracket \phi \rrbracket = \llbracket \psi \rrbracket \\ 1 - \llbracket \psi \rrbracket + \llbracket \phi \rrbracket \text{ if } \llbracket \phi \rrbracket < \llbracket \psi \rrbracket \\ 1 - \llbracket \phi \rrbracket + \llbracket \psi \rrbracket \text{ if } \llbracket \phi \rrbracket > \llbracket \psi \rrbracket \end{cases}$	$\begin{cases} 1 \text{ if } \llbracket \phi \rrbracket = \llbracket \psi \rrbracket \\ \llbracket \phi \rrbracket \text{ if } \llbracket \phi \rrbracket < \llbracket \psi \rrbracket \\ \llbracket \psi \rrbracket \text{ if } \llbracket \phi \rrbracket > \llbracket \psi \rrbracket \end{cases}$	$\begin{cases} 1 \text{ if } \llbracket \phi \rrbracket = \llbracket \psi \rrbracket \\ \llbracket \phi \rrbracket / \llbracket \psi \rrbracket \text{ if } \llbracket \phi \rrbracket < \llbracket \psi \rrbracket \\ \llbracket \psi \rrbracket / \llbracket \phi \rrbracket \text{ if } \llbracket \phi \rrbracket > \llbracket \psi \rrbracket \end{cases}$

In Łukasiewicz Logic, the residual conditional is also a material conditional, and the residual biconditional is a material biconditional. Otherwise, the three types of conditional are distinct. The following table shows that the residual, material and probabilistic conditionals generally exhibit distinct truth degrees.

(52)	Three (Bi)conditionals	*Łukasiewicz Logic*		*Gödel Logic*		*Product Logic*	
	Component Formulas:	$\llbracket \phi \rrbracket$ $\llbracket \psi \rrbracket$	$\llbracket \phi \rrbracket$ $\llbracket \psi \rrbracket$	$\llbracket \phi \rrbracket$ $\llbracket \psi \rrbracket$	$\llbracket \phi \rrbracket$ $\llbracket \psi \rrbracket$	$\llbracket \phi \rrbracket$ $\llbracket \psi \rrbracket$	$\llbracket \phi \rrbracket$ $\llbracket \psi \rrbracket$
	Values:	26% 75%	75% 26%	75% 26%	67% 67%	26% 75%	75% 26%
a.	Residual: $\llbracket \phi \rightarrow \psi \rrbracket$	100%	51%	26%	100%	100%	35%
b.	Residual: $\llbracket \phi \leftrightarrow \psi \rrbracket$	51%	51%	26%	100%	35%	35%
c.	$\llbracket \neg\phi \vee \psi \rrbracket$	$= \llbracket \phi \rightarrow \psi \rrbracket$		26%	67%	$\llbracket \psi \rrbracket$	
d.	$\llbracket (\neg\phi \vee \psi) \wedge (\phi \vee \neg\psi) \rrbracket$	$= \llbracket \phi \leftrightarrow \psi \rrbracket$		26%	67%	20%	20%
e.	$\llbracket \phi \wedge \psi \rrbracket / \llbracket \phi \rrbracket \text{ if } \llbracket \phi \rrbracket > 0$	4%	1%	35%	100%	$\llbracket \psi \rrbracket$	
f.	$\begin{cases} \llbracket \psi \rrbracket / \llbracket \phi \rrbracket \text{ if } 0 < \llbracket \psi \rrbracket \leq \llbracket \phi \rrbracket \\ \llbracket \phi \rrbracket / \llbracket \psi \rrbracket \text{ if } 0 < \llbracket \phi \rrbracket < \llbracket \psi \rrbracket \end{cases}$	35%	35%	35%	100%	$\llbracket \phi \leftrightarrow \psi \rrbracket$	

Item C (Section 2.2)
The information on historical yield inversions and recessions is taken from https://www.investopedia.com and from Pan (2006).

Yield Curve Inversion	Recession in USA	Yield Curve Inversion	Recession in USA
---	1937 The Roosevelt Recession	1980	1981 The Iran Crisis Recession
---	1945 The Union Recession	1982	---
---	1948 The Post-War Recession	1988	---
---	1953 The Post-Korean War Recession	1989	1990 The Gulf War Recession
1956	1957 The Eisenhower Recession	1998	---
1959	1960 The “Rolling Adjustment” Recession	2000	2001 The 9/11 Recession
1965	---	2006	2008 The Great Recession
1969	1969 The Nixon Recession	2019	
1972	1973 The Oil Crisis Recession		
1978	1980 The Energy Crisis Recession		

Table 3: Yield Curve inversions and recessions in the USA since 1937

Item D (Section 2.2)
The truth degree of χ (*There will be a pandemic next year*) crucially relies on data available in 2019. Table 4, presenting a daily breakdown of cases, deaths, and discharges for Wuhan in December 2019, is derived from a WHO-China document published by the *World Health Organization* on 28-February-2020 (WHO 2020). Although the cases are included in this document (p.6), the death and discharge numbers are extrapolated from two pieces of information reported on p. 12 and p.14: Cases have an outcome after approximately 17 days of infection, with the crude fatality ratio on 20-February-2020 being 3.8%.

Day	Cases	Deaths	Recoveries	Day	Cases	Deaths	Recoveries	Day	Cases	Deaths	Recoveries	Day	Cases	Deaths	Recoveries
02	1	0	0	10	1	0	0	18	4	0	0	26	11	0	2
03	0	0	0	11	2	0	0	19	3	0	1	27	29	0	2
04	0	0	0	12	4	0	0	20	12	0	0	28	13	0	4
05	0	0	0	13	0	0	0	21	5	0	0	29	17	0	0
06	0	0	0	14	0	0	0	22	10	0	0	30	21	0	0
07	0	0	0	15	4	0	0	23	12	0	0	31	18	1	3
08	1	0	0	16	2	0	0	24	12	0	1	Total:	203	1	13
09	0	0	0	17	9	0	0	25	12	0	0				

Table 4: Covid-19 Index Case and course of first thirty days in Wuhan (December 2019)

In mathematical epidemiology, the SIR Model (*Susceptible-Infected-Removed Model*) uses these initial data to predict the future course of a contagious disease (Hethcote 1989). In particular, health officials employ these data to set, or rather to approximate, the parameters $\boldsymbol{\beta}$ (the number of people an infective person transmits the disease to on a daily basis) and $\boldsymbol{\gamma}$ (the proportion of infected people who die or recover from the disease on a daily basis). Every contagious disease splits a population of size N (Wuhan 11,9M) into three compartments on any given day (t) of its course:

- $S(t)$: the number of susceptible individuals on day t who can be but are not yet infected;
- $I(t)$: the number of infected individuals who are transmitting the disease on day t;
- $R(t)$: the number of individuals who are removed on day t from the susceptible/infective interaction by recovery, isolation, or death.

Let $s(t) = S(t)/N$, $i(t) = I(t)/N$ and $r(t) = R(t)/N$ be the population fractions of these three groups, for which we have $s(t) + i(t) + r(t) = 1$. The three functions $s(t)$, $i(t)$ and $r(t)$ are the unique solutions of an *initial value problem* (IVP), that is, of three differential equations with three initial conditions:

Linear Differential Equations of first order:	*Initial Conditions:*	*Wuhan Initial Conditions:*[25]
- $s'(t) = -\boldsymbol{\beta}\cdot s(t)\cdot i(t)$	- $s(t_0) = s_0$	- $s(30) = 0.99998294$
- $i'(t) = \boldsymbol{\beta}\cdot s(t)\cdot i(t) - \boldsymbol{\gamma}\cdot i(t)$	- $i(t_0) = i_0$	- $i(30) = 0.00001588$
- $r'(t) = \boldsymbol{\gamma}\cdot i(t)$	- $r(t_0) = r_0$	- $r(30) = 0.00000117$

Given that the first 30 days in Wuhan provide the blueprint of Covid-19, the initial conditions for this pandemic are derived from the figures[35] on the 30th day divided by the population size of 11.9M. The standard numerical method for approximating solutions of differential equations involves Euler series. Here, we use the first 30 members of the Euler series for approximating **β** and **γ**:

Euler series for the SIR Model:	*First tangent point:*	*Second tangent point:*
- $s(t_{n+1}) = s(t_n) - \boldsymbol{\beta}\cdot s(t_n)\cdot i(t_n)$	- $s(t_{-29}) = 0.99999991$	- $s(t_0) = 0.99998294$
- $i(t_{n+1}) = i(t_n) + \boldsymbol{\beta}\cdot s(t_n)\cdot i(t_n) - \boldsymbol{\gamma}\cdot i(t_n)$	- $i(t_{-29}) = 0.00000008$	- $i(t_0) = 0.00001588$
- $r(t_{n+1}) = r(t_n) + \boldsymbol{\gamma}\cdot i(t_n)$	- $r(t_{-29}) = 0$	- $r(t_0) = 0.00000117$

I have written a computer program that approximates the Covid-19 transmission rate and removal rate as $\boldsymbol{\beta} = 0.21286905$ and $\boldsymbol{\gamma} = 0.014752875$ (with an error margin of $\approx 10^{-9}$). Simply put, these rates mean that when no lockdown narrowed down the number of susceptible individuals during the first 30 days of Covid-19, every infected person transmitted the disease to about 0.21 susceptible people per day, while roughly 1% of the infected people recovered or died from the disease on a daily basis. Now if the *initial replacement number* ρ_0 of a disease, as defined below, is greater than one, then epidemiologists regard the disease as an epidemic; if ρ_0 is equal to one or smaller, then the disease is not viewed as an epidemic. Furthermore, the formula given below can be used to calculate the maximal proportion of the infected class.

Initial disease replacement number:		*Maximal proportion of infected people in an epidemic:*	
General (Hethcote 1989:123)	Covid-19 (Wuhan)	General (Hethcote 1989:128)	Covid-19 (Wuhan)
$\rho_0 = \frac{\boldsymbol{\beta}}{\boldsymbol{\gamma}} \cdot s_0$	$\rho_0 = 14.4287415$	$I_{\max} = 1 - r_0 - \frac{\boldsymbol{\gamma}}{\boldsymbol{\beta}} - \frac{\boldsymbol{\gamma}}{\boldsymbol{\beta}} \cdot \log_e\left(\frac{\boldsymbol{\beta}}{\boldsymbol{\gamma}} \cdot s_0\right)$	$I_{\max} = 0.745703639$

As the replacement number of Covid-19 was about 14.4287415 on the last day of 2019, it can be inferred that based on the numbers shown in Table 4, a serious epidemic loomed in the Wuhan area for 2020 that threatened to contaminate 75% of the local population at its peak. In order to make a prediction on the last day of 2019 whether a pandemic loomed in 2020, we need to access a database of historical contagious diseases such as the ca. 250 entries of Byrne (2008)'s *Encyclopedia of Pandemics and Plagues*, which includes the following known events.

[35] From Table 4, we can derive: $S(30) = 11.899.797$; $I(30) = 189$; $R(30) = 14$.

Name	*Time period*	*Type / Pre-human host*	*Death toll*
Antonine Plague	165-180	Believed to be smallpox or measles	5M
Japanese smallpox	735-737	Variola major virus	1M
Plague of Justinian	541-542	Yersinia pestis bacteria / Rats, fleas	30-50M
Black Death	1347-1351	Yersinia pestis bacteria / Rats, fleas	200M
New World Smallpox	1520 – onwards	Variola major virus	56M
Italian plague	1629-1631	Yersinia pestis bacteria / Rats, fleas	1M
Cholera Pandemics 1-6	1817-1923	V. cholerae bacteria	1M+
Third Plague	1885	Yersinia pestis bacteria / Rats, fleas	12M
Russian Flu	1889-1890	Believed to be H2N2 (avian origin)	1M
Spanish Flu	1918-1919	H1N1 virus / Pigs	40-50M
Asian Flu	1957-1958	H2N2 virus	1.1M
Hong Kong Flu	1968-1970	H3N2 virus	1M
HIV/AIDS	1981-present	Virus / Chimpanzees	25-35M

Table 5: Historical pandemics (Byrne 2008)

Although the genome of some past pandemic viruses and bacteria has been identified, the reproduction and recovery rates (**β** and **γ**) of almost all past diseases are unavailable. Suppose that 11 of the 250 contagious diseases mentioned in Byrne's encyclopedia show similar rates (**β** and **γ**) as Covid-19. Thus, these 11 diseases represent the sample **Σ** of the pandemic prediction. Suppose that nine of these diseases (**X**) have led to one of the pandemics listed in Table 5. Therefore, the prediction χ (*There will be a pandemic next year*) has the truth degree **P** = 9/11 = 82%.

Item E (Section 2.2)
The sample and property set of (9c-f), relabeled as (53c-f),

(53) c. Bimbo will neither hike nor diminish its bakery business next year.
d. Clorox will strongly increase sales of its disinfecting products next year.
e. The car rental company Hertz will file for bankruptcy next year.
f. Kimberley-Clark will greatly boost sales of its paper towels next year.

are cartesian products or arrayed data structures (as used in relational SQL databases). A financial analytic firm holds its data in the following kind of table which includes entries going back to 1913 when the first disinfecting companies and rental car companies were founded.

Field Name	Data Type	Description
Company	Text	Name of Company
Year	Number	Year in the past to which 2019 is compared
Competitors	Text	Competitors of the company in 2019
Partners	Text	Partners of the company in 2019
Event	Number	1 = epidemic; 2 = war; 3 = recession; 4 = other disaster; 5 = no event in following year
Profitability	Number	Change in profit margins (%) in the year of comparison
Bankruptcy Year+1	Yes/No	Did bankruptcy occur in year following the year of comparison?
Sales Change Year+1	Number	Increase/decrease of sales (%) in year following the year of comparison?

Table 6: Datasheet of stock exchange-listed companies (Example)

The samples are drawn from this table by applying filters (queries), while the property sets are derived from the samples by using additional filters. Table 7 provides hypothetical cardinalities for each set. For example, the sample $\mathbf{\Sigma}_\phi$ consists of those competitors or partners of Clorox that in a past year experienced an epidemic or were approximately as profitable as Clorox. The property set $\mathbf{X}_\phi$ comprises of those companies in the sample that increased their sales by at least 9% in the following year. In addition to the premise and filters of $\mathbf{\Sigma}_\phi$, $\mathbf{X}_\phi$ thus uses one additional microeconomic filter, i.e. [Sales In/Decrease Year+1] $\geq$ 9%.

	Premise	Macroeconomic filter	Microeconomic filter	Cardinality
Σ:	Bimbo ∈ [Competitors] ∪ [Partners]	∧ ([Event] = “1,2,4”	∨ [Profitability] ≈ Bimbo profitability 2019)	523
X:			[Sales In/Decrease Year+1] ≈ 0%	214
Σ:	Clorox ∈ [Competitors] ∪ [Partners]	∧ ([Event] = “1”	∨ [Profitability] ≈ Clorox profitability 2019)	349
X:			[Sales In/Decrease Year+1] ≥ 9%	73
Σ:	Hertz ∈ [Competitors] ∪ [Partners]	∧ ([Event] = “1,2,3,4”	∨ [Profitability] ≈ Hertz profitability 2019)	216
X:			[Bankruptcy Year+1] = “Yes”	5
Σ:	Kimberley-Clark∈ [Competitors] ∪ [Partners]	∧ ([Event] = “1,2,4”	∨ [Profitability] ≈ KC profitability 2019)	421
X:			[Sales In/Decrease Year+1] ≥ 9%	80

Table 7: Sample and property set of four economic predictions

Item F (Section 2.5)
Below is a hypothetical example of a population (sample) of n = 27 owls with their respective number of hunts u_k (k = 1,…,27) during a three-months trial period. All actual hunts lie below q = 200, the maximal number of possible hunting events. Five baby owls did not hunt any mouse (thus m = 22). The ornithologists decided that an owl must hunt r = 30 mice in 180 days to count as prey bird.

Owl ID	Hunts	Owl ID	Hunts	Owl ID	Hunts	Owl ID	Hunts	Owl ID	Hunts	Owl ID	Hunts
1	0	6	10	11	54	16	85	21	108	26	149
2	0	7	18	12	58	17	88	22	117	27	151
3	0	8	24	13	65	18	92	23	122		
4	0	9	31	14	73	19	97	24	135		
5	0	10	39	15	81	20	101	25	146		

Table 8: Number of hunts per owl during 180 days

Item G (Section 2.6)
The height of all players of Dallas Mavericks and of Utah Jazz are juxtaposed in the following table and allow to establish the cardinal number of $\mathbf{X} = \{(x_1, x_2) \mid x_1 > x_2\}$ as 162.

Σ_1 = Dallas Mavericks			Σ_2 = Utah Jazz		
Height cm	Height cm	Height cm	Height cm	Height cm	Height cm
178	196	208	183	193	201
185	196	208	185	193	203
188	198	213	185	196	206
193	201	221	188	196	208
196	201	224	190	201	216
196	201		190	201	

Table 9: Comparison of Dallas Mavericks and Utah Jazz

Item H (Section 2.6)
If we denote Σ_1 = 'Chicago Bulls' and Σ_2 = 'NBA Players' without 'Chicago Bulls', then the number of positive comparisons $X = \{(x_1, x_2) \mid x_1 > x_2\}$ can be obtained by adding up the numbers of the second, fourth and sixth column, which is 4152.

Height of Σ_1 player cm	Number of smaller Σ_2 players	Height of Σ_1 player cm	Number of smaller Σ_2 players	Height of Σ_1 player cm	Number of smaller Σ_2 players
190	57	198	192	206	347
190	57	201	247	208	385
193	99	201	247	208	385
193	99	201	247	213	450
196	137	203	296	218	474
196	137	203	296		

Table 10: Comparison of 'Chicago Bulls' and 'NBA Players without Chicago Bulls'

Item I (Section 2.6)
Let us write Σ_1 = 'Dallas Mavericks' and Σ_2 = **Lower Half** of 'NBA Players' without 'Dallas Mavericks', then the number of positive comparisons can be added up from the numbers in the second, fourth and sixth column. It is 2816. (Upper/lower half refers to the median value that splits a distribution into two halves.)

Height of Σ_1 player cm	Number of smaller Σ_2 players	Height of Σ_1 player cm	Number of smaller Σ_2 players	Height of Σ_1 player cm	Number of smaller Σ_2 players
178	1	196	137	208	240
185	18	196	137	208	240
188	41	198	190	213	240
193	98	201	240	221	240
196	137	201	240	224	240
196	137	201	240		

Table 11: Comparison of 'Dallas Mavericks' and 'NBA Players without Dallas Mavericks'

Item J (Section 3.1)
In this subsection I show that $[\![\cdot]\!]$: SENT $\rightarrow$ [0,1] in Sample Logic satisfies the slightly modified Kolmogorov axioms of (20) relabeled as (54).

(54) *Proof*:

K1 The truth degree is between 0 and 1, because the property set is a subset of the sample.

K2 The formulas $\phi \wedge \neg\phi$ and $\phi \vee \neg\phi$ are evaluated according to (19ciii). By design, we have $\mathbf{X}_\phi \cap \mathbf{X}_{\neg\phi} = \varnothing$ and $\mathbf{X}_\phi \cup \mathbf{X}_{\neg\phi} = \mathbf{\Sigma}_\phi$ and thus $[\![\phi \wedge \neg\phi]\!] = 0$ and $[\![\phi \vee \neg\phi]\!] = 1$;

K3' We show axiom K3 for independent propositions and propositions with $\mathbf{\Sigma}_\phi = \mathbf{\Sigma}_\psi$. We assume that $[\![\phi \wedge \psi]\!] = 0$. If ϕ and ψ are independent, then $\mathbf{\Sigma}_\phi$ and $\mathbf{\Sigma}_\psi$ are uncorrelated and we have $\mathbf{X}_{\phi\wedge\psi} = \mathbf{X}_\phi \times \mathbf{X}_\psi = \varnothing$, thus either $\mathbf{X}_\phi = \varnothing$ or $\mathbf{X}_\psi = \varnothing$.
If $\mathbf{X}_\phi = \varnothing$ then $\mathbf{X}_{\phi\vee\psi} = \mathbf{\Sigma}_\phi \times \mathbf{X}_\psi$ and $[\![\phi \vee \psi]\!] = [\![\phi]\!] + [\![\psi]\!]$; if $\mathbf{X}_\psi = \varnothing$, then $\mathbf{X}_{\phi\vee\psi} = \mathbf{X}_\phi \times \mathbf{\Sigma}_\psi$ and also $[\![\phi \vee \psi]\!] = [\![\phi]\!] + [\![\psi]\!]$.
In the second case, let us suppose that $\mathbf{\Sigma}_\phi = \mathbf{\Sigma}_\psi$ and $[\![\phi \wedge \psi]\!] = 0$. According to (19c-iii), we have $\mathbf{X}_{\phi\wedge\psi} = \mathbf{X}_\phi \cap \mathbf{X}_\psi = \varnothing$. It follows that $|\mathbf{X}_{\phi\vee\psi}| = |\mathbf{X}_\phi \cup \mathbf{X}_\psi| = |\mathbf{X}_\phi| + |\mathbf{X}_\psi|$ and hence $[\![\phi \vee \psi]\!] = [\![\phi]\!] + [\![\psi]\!]$.

K4 In the first case, let ϕ and ψ be independent; $\mathbf{\Sigma}_\phi$ and $\mathbf{\Sigma}_\psi$ are uncorrelated and we have $\mathbf{X}_{\phi\vee\psi}$ = ($\mathbf{X}_\phi \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_{\neg\phi} \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_\phi \times \mathbf{X}_{\neg\psi}$) = ($\mathbf{\Sigma}_\phi \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_\phi \times \mathbf{X}_{\neg\psi}$) ⊂ ($\mathbf{\Sigma}_\phi \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_\phi \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_\phi \times \mathbf{X}_{\neg\psi}$) = ($\mathbf{\Sigma}_\phi \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_\phi \times \mathbf{\Sigma}_\psi$). It follows that

$$[\![\phi \vee \psi]\!] = \frac{|\mathbf{X}_{\phi\vee\psi}|}{|\mathbf{\Sigma}_{\phi\vee\psi}|} \leq \frac{|\mathbf{\Sigma}_\phi| \cdot |\mathbf{X}_\psi| + |\mathbf{X}_\phi| \cdot |\mathbf{\Sigma}_\psi|}{|\mathbf{\Sigma}_\phi \times \mathbf{\Sigma}_\psi|} = \frac{|\mathbf{\Sigma}_\phi| \cdot |\mathbf{X}_\psi|}{|\mathbf{\Sigma}_\phi| \cdot |\mathbf{\Sigma}_\psi|} + \frac{|\mathbf{X}_\phi| \cdot |\mathbf{\Sigma}_\psi|}{|\mathbf{\Sigma}_\phi| \cdot |\mathbf{\Sigma}_\psi|} = [\![\phi]\!] + [\![\psi]\!]$$

In the second case, let ϕ and ψ be dependent. As $\mathbf{\Sigma}_\phi$ and $\mathbf{\Sigma}_\psi$ are correlated, the following relations hold: $[\![\phi \vee \psi]\!] = \frac{|\mathbf{X}_{\phi\vee\psi}|}{|\mathbf{\Sigma}_{\phi\vee\psi}|} = \frac{|\mathbf{X}_\phi \cup \mathbf{X}_\psi|}{|\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi|} \leq \frac{|\mathbf{X}_\phi| + |\mathbf{X}_\psi|}{|\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi|} =$

$$\frac{|\mathbf{X}_\phi|}{|\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi|} + \frac{|\mathbf{X}_\psi|}{|\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi|} \leq \frac{|\mathbf{X}_\phi|}{|\mathbf{\Sigma}_\phi|} + \frac{|\mathbf{X}_\psi|}{|\mathbf{\Sigma}_\psi|} = [\![\phi]\!] + [\![\psi]\!].$$

Item K (Section 3.1)
We demonstrate for Sample Logic in this subsection that $[\![\neg\phi]\!] = 1 - [\![\phi]\!]$, that $[\![\cdot]\!]$ is idempotent and coherent, and that the two de Morgan laws hold.

(55) *Proof*:

a. We have $[\![\neg\phi]\!] = 1 - [\![\phi]\!]$ which directly follows from the definition of negation in (17).

b. In order to show that $[\![\phi \wedge \phi]\!] = [\![\phi]\!]$, the property set $\mathbf{X}_{\phi\wedge\phi}$ is $\mathbf{X}_\phi \cap \mathbf{X}_\phi = \mathbf{X}_\phi$ according to the definition (19ciii). The evaluation is thus idempotent.

c. In the same vein, the property set $\mathbf{X}_{\phi\wedge\neg\phi}$ is $\mathbf{X}_\phi \cap \mathbf{X}_{\neg\phi} = \varnothing$ according to the definition (19ciii). The evaluation is thus coherent.

d. To demonstrate the de Morgan laws, we first consider the case where ϕ and ψ are independent. According to (19b), we have
$\mathbf{X}_{\phi\wedge\psi} \cup \mathbf{X}_{\neg\phi\vee\neg\psi}$ = ($\mathbf{X}_\phi \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_{\neg\phi} \times \mathbf{X}_{\neg\psi}$) ∪ ($\mathbf{X}_\phi \times \mathbf{X}_{\neg\psi}$) ∪ ($\mathbf{X}_{\neg\phi} \times \mathbf{X}_\psi$) = $\mathbf{\Sigma}_\phi \times \mathbf{\Sigma}_\psi$ and
$\mathbf{X}_{\phi\wedge\psi} \cap \mathbf{X}_{\neg\phi\vee\neg\psi} = \varnothing$. If follows thus $[\![\neg(\phi \wedge \psi)]\!] = [\![\neg\phi \vee \neg\psi]\!]$. Furthermore, we have
$\mathbf{X}_{\phi\vee\psi} \cup \mathbf{X}_{\neg\phi\wedge\neg\psi}$ = ($\mathbf{X}_\phi \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_{\neg\phi} \times \mathbf{X}_\psi$) ∪ ($\mathbf{X}_\phi \times \mathbf{X}_{\neg\psi}$) ∪ ($\mathbf{X}_{\neg\phi} \times \mathbf{X}_{\neg\psi}$) = $\mathbf{\Sigma}_\phi \times \mathbf{\Sigma}_\psi$ and
$\mathbf{X}_{\phi\vee\psi} \cap \mathbf{X}_{\neg\phi\wedge\neg\psi} = \varnothing$. We can conclude that $[\![\neg(\phi \vee \psi)]\!] = [\![\neg\phi \wedge \neg\psi]\!]$.

In the second case, let ϕ and ψ coordinated. According to (19c), we have
$\mathbf{X}_{\phi\wedge\psi} \cup \mathbf{X}_{\neg\phi\vee\neg\psi}$ = ($\mathbf{X}_\phi \cap \mathbf{X}_\psi$) ∪ ($\mathbf{X}_\phi \setminus \mathbf{\Sigma}_\psi$) ∪ ($\mathbf{X}_\psi \setminus \mathbf{\Sigma}_\phi$) ∪ ($\mathbf{X}_{\neg\phi} \cup \mathbf{X}_{\neg\psi}$) = $\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi$ and
$\mathbf{X}_{\phi\wedge\psi} \cap \mathbf{X}_{\neg\phi\vee\neg\psi} = \varnothing$. If follows thus $[\![\neg(\phi \wedge \psi)]\!] = [\![\neg\phi \vee \neg\psi]\!]$. In addition, we also have
$\mathbf{X}_{\phi\vee\psi} \cup \mathbf{X}_{\neg\phi\wedge\neg\psi}$ = ($\mathbf{X}_\phi \cup \mathbf{X}_\psi$) ∪ ($\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi}$) ∪ ($\mathbf{X}_{\neg\phi} \setminus \mathbf{\Sigma}_\psi$) ∪ ($\mathbf{X}_{\neg\psi} \setminus \mathbf{\Sigma}_\phi$) = $\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi$ and
$\mathbf{X}_{\phi\vee\psi} \cap \mathbf{X}_{\neg\phi\wedge\neg\psi} = \varnothing$. If follows thus that $[\![\neg(\phi \vee \psi)]\!] = [\![\neg\phi \wedge \neg\psi]\!]$.

In the third case, let ψ be subordinated to ϕ. If $(b_1,\ldots b_n) \in \mathbf{\Sigma}_{\phi\vee\psi}$, then according to (19d), there is $(a_1,\ldots,a_i) \in \mathbf{\Sigma}_\phi$ with $a_1 = b_1,\ldots, a_i = b_i$. If $(a_1,\ldots,a_i) \in \mathbf{X}_\phi$, then $(b_1,\ldots b_n) \in \mathbf{X}_{\phi\wedge\psi}$ or $(b_1,\ldots b_n) \in \mathbf{X}_{\phi\wedge\neg\psi} \subseteq \mathbf{X}_{\neg\phi\vee\neg\psi}$; if $(a_1,\ldots,a_i) \in \mathbf{X}_{\neg\phi}$, then $(b_1,\ldots b_n) \in \mathbf{X}_{\neg\phi\vee\neg\psi}$, and therefore $\mathbf{X}_{\phi\wedge\psi} \cup \mathbf{X}_{\neg\phi\vee\neg\psi} = \mathbf{\Sigma}_{\phi\vee\psi}$. Furthermore, let $(b_1,\ldots b_n) \in \mathbf{X}_{\phi\wedge\psi}$. According to (19d), there is $(a_1,\ldots,a_i) \in \mathbf{X}_\phi$ with $a_1 = b_1,\ldots, a_i = b_i$. There cannot be $(c_1,\ldots,c_j) \in \mathbf{X}_{\neg\phi}$ with $c_1 = b_1,\ldots, c_j = b_j$. We therefore conclude $\mathbf{X}_{\phi\wedge\psi} \cap \mathbf{X}_{\neg\phi\vee\neg\psi} = \varnothing$ and $[\![\neg(\phi \wedge \psi)]\!] = [\![\neg\phi \vee \neg\psi]\!]$. On the other hand, if $(b_1,\ldots b_n) \in \mathbf{\Sigma}_{\phi\vee\psi}$, then there is $(a_1,\ldots,a_i) \in \mathbf{\Sigma}_\phi$ with $a_1 = b_1,\ldots, a_i = b_i$.
If $(a_1,\ldots,a_i) \in \mathbf{X}_\phi$, then $(b_1,\ldots b_n) \in \mathbf{X}_{\phi\vee\psi}$; if $(a_1,\ldots,a_i) \in \mathbf{X}_{\neg\phi}$, then $(b_1,\ldots b_n) \in \mathbf{X}_{\neg\phi\wedge\neg\psi}$ or $(b_1,\ldots b_n) \in \mathbf{X}_{\phi\vee\psi}$. Moreover, let $(b_1,\ldots b_n) \in \mathbf{X}_{\neg\phi\wedge\neg\psi}$. According to (19d), there is $(a_1,\ldots,a_i) \in \mathbf{X}_{\neg\phi}$ with $a_1 = b_1,\ldots, a_i = b_i$. There cannot be $(c_1,\ldots,c_j) \in \mathbf{X}_\phi$ with $c_1 = b_1,\ldots, c_j = b_j$. It follows that $\mathbf{X}_{\phi\vee\psi} \cap \mathbf{X}_{\neg\phi\wedge\neg\psi} = \varnothing$ and $[\![\neg(\phi \vee \psi)]\!] = [\![\neg\phi \wedge \neg\psi]\!]$.

Item L (Section 3.2)

The sample and property set of the conjunction of ϕ (*Clorox will strongly increase sales of its disinfecting products*) and ψ (*Kimberley-Clark will greatly boost sales of its paper towels next year*) can be derived from table 7 and from table 12 below. (Each time the property sets share the premises and filters of the sample and are filtered by one additional microeconomic property.) The truth conjunctive degree is $[\![\phi \wedge \psi]\!] = 17\% = 107 / 627$.

	Premise	Macroeconomic filter	Microeconomic filter	Cardinality
$\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi$:	Clorox or KC$\in$[Competitors] $\cup$ [Partners]	$\wedge$ ([Event] = "1,2,4"	$\vee$ [Profitability] $\approx$ Clorox or KC profitability 2019)	627
$\mathbf{X}_\phi \cap \mathbf{X}_\psi$:			[Sales In/Decrease Year+1] $\geq$ 9%	31
$\mathbf{\Sigma}_\phi \setminus \mathbf{\Sigma}_\psi$:	Clorox$\in$, KC$\notin$[Competitors] $\cup$ [Partners]	$\wedge$ ([Event] = "1"	$\vee$ [Profitability] $\approx$ Clorox profitability 2019)	206
$\mathbf{X}_\phi \setminus \mathbf{\Sigma}_\psi$:			[Sales In/Decrease Year+1] $\geq$ 9%	33
$\mathbf{\Sigma}_\psi \setminus \mathbf{\Sigma}_\phi$:	Clorox$\notin$, KC$\in$[Competitors] $\cup$ [Partners]	$\wedge$ ([Event] = "1,4"	$\vee$ [Profitability] $\approx$ KC profitability 2019)	278
$\mathbf{X}_\psi \setminus \mathbf{\Sigma}_\phi$:			[Sales In/Decrease Year+1] $\geq$ 9%	43
$\mathbf{\Sigma}_{\phi\wedge\psi}$:	$\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi$			627
$\mathbf{X}_{\phi\wedge\psi}$:	$(\mathbf{X}_\phi \cap \mathbf{X}_\psi) \cup (\mathbf{X}_\phi \setminus \mathbf{\Sigma}_\psi) \cup (\mathbf{X}_\psi \setminus \mathbf{\Sigma}_\phi)$			107

Table 12: Sample and property set of the conjunction $\phi\wedge\psi$

Item M (Section 3.2)

The truth degree of the conjunction of ϕ (*There is a pandemic next year*) and ψ (*Clorox will strongly increase sales of its disinfecting products*) can be derived from table 13 which applies $\mathbf{\Sigma}_\phi$ as a filter onto $\mathbf{\Sigma}_\psi$ (see table 7, item E). The conjunctive truth degree is $[\![\phi \wedge \psi]\!] = 71\% = 68/96$.

	Premise	Macroeconomic filter	Microeconomic filter	Cardinality
$\mathbf{\Sigma}_{\phi\wedge\psi}$:	Clorox $\in$ [Competitors] $\cup$ [Partners] $\wedge$	([Event] = "1")		96
$\mathbf{X}_{\phi\wedge\psi}$:		([Event] = "pandemic" $\wedge$	[Sales In/Decrease Year+1] $\geq$ 9%	68
$\mathbf{X}_{\neg\phi\wedge\psi}$:		([Event] = "$\neg$pandemic" $\wedge$	[Sales In/Decrease Year+1] $\geq$ 9%	4
$\mathbf{X}_{\phi\wedge\neg\psi}$:		([Event] = "pandemic" $\wedge$	[Sales In/Decrease Year+1] < 9%	2
$\mathbf{X}_{\neg\phi\wedge\neg\psi}$:		([Event] = "$\neg$pandemic" $\wedge$	[Sales In/Decrease Year+1] < 9%	22

Table 13: Sample and property of Clorox performance under pandemic

Item N (Section 3.2)

The truth degree of the conjunction of ϕ (*The Dallas Mavericks players are taller than the Utah Jazz players*) and ψ (*The Dallas Mavericks players are taller than the Denver Nuggets players*) can be computed from the figures in table 9 (item G) above (ϕ) and those in table 14 below (ψ). We can establish the cardinal number of $\mathbf{X}_\psi = \{(x_1, x_2) \mid x_1 > x_2\}$ as 108.

Σ_1 = Dallas Mavericks			Σ_2 = Denver Nuggets		
Height cm	Height cm	Height cm	Height cm	Height cm	Height cm
178	196	208	188	201	208
185	196	208	193	201	211
188	198	213	193	203	213
193	201	221	193	203	218
196	201	224	196	203	
196	201		198	208	

Table 14: Comparison of Dallas Mavericks and Denver Nuggets

The conjunctive truth degree is calculated according to (19c-i) by following the steps in the following table.

Sample:	Cardinality	Property set:	Cardinality
$\mathbf{\Sigma}_\phi$ = {Heights of Dallas Mavericks} × {Heights of Utah Jazz}	289	$\mathbf{X}_\phi = \{(x_1,x_2) \in \mathbf{\Sigma}_\phi \mid x_1 > x_2\}$	162
$\mathbf{\Sigma}_\psi$ = {Heights of Dallas Mavericks} × {Heights of Denver Nuggets}	272	$\mathbf{X}_\psi = \{(x_1,x_2) \in \mathbf{\Sigma}_\psi \mid x_1 > x_2\}$	108
$\mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi$ = {Heights of Dallas Mavericks} × {Heights of UJ and DN}	561	$\mathbf{X}_\phi \cup \mathbf{X}_\psi = \{(x_1,x_2) \in \mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi \mid x_1 > x_2\}$	270
$\mathbf{\Sigma}_{\phi\wedge\psi} = \mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi$	561	$\mathbf{X}_{\phi\wedge\psi} = \mathbf{X}_\phi \cup \mathbf{X}_\psi$	270

Table 15: Sample and property set of the conjunction $\phi\wedge\psi$ with $\mathbf{\Sigma}_\phi \cap \mathbf{\Sigma}_\psi = \varnothing$

Item O (Section 3.2)

The truth degree of the conjunction of ϕ (*The Dallas Mavericks players are taller than the Utah Jazz players*) and ψ (*The Dallas Mavericks players are not the tallest team of the NBA*) can be computed from the figures in table 9 (item G) above (ϕ) and those in table 16 below (ψ). For determining the property set of $\neg\psi$ (*The players of Dallas Mavericks are the tallest team of the NBA*), let us write Σ_1 = 'Dallas Mavericks' and Σ_2 = 'NBA Players' without 'Dallas Mavericks'. The number of positive comparisons $\mathbf{X}_{\neg\psi} = \{(x_1, x_2) \mid x_1 > x_2\}$ can be obtained by adding up the numbers of the second, fourth and sixth column. Its cardinal number is 3812.

Height of Σ_1 player cm	Number of smaller Σ_2 players	Height of Σ_1 player cm	Number of smaller Σ_2 players	Height of Σ_1 player cm	Number of smaller Σ_2 players
178	1	196	137	208	386
185	18	196	137	208	386
188	41	198	190	213	451
193	98	201	245	221	479
196	137	201	245	224	479
196	137	201	245		

Table 16: Comparison of 'Dallas Mavericks' and 'NBA Players without Dallas Mavericks'

The property set $\mathbf{X}_\psi$ thus counts (17×480) - 3812 = 4348 positive comparisons. The truth degree of $\phi\wedge\psi$ can be calculated as 52% (4221/8160) by following the steps explicated in table 17.

Sample:	Cardinality	Property set:	Cardinality
$\mathbf{\Sigma}_\phi$ = {Dallas Mavericks height} × {Utah Jazz height}	289	$\mathbf{X}_\phi = \{(x_1,x_2) \in \mathbf{\Sigma}_\phi \mid x_1 > x_2\}$	162
$\mathbf{\Sigma}_\psi$ = {DM height} × {Height of NBA without DM}	8160	$\mathbf{X}_\psi = \{(x_1,x_2) \in \mathbf{\Sigma}_\psi \mid x_1 \leq x_2\}$	4348
		$\mathbf{X}_\phi \cap \mathbf{X}_\psi = \{(x_1,x_2) \in \mathbf{\Sigma}_\phi \mid x_1 > x_2 \text{ and } x_1 \leq x_2\}$	0
$\mathbf{\Sigma}_\psi \setminus \mathbf{\Sigma}_\phi$ = {DM height} × {Height of NBA without DM and UJ}	7871	$\mathbf{X}_\psi \setminus \mathbf{\Sigma}_\phi = \{(x_1,x_2) \in \mathbf{\Sigma}_\psi \setminus \mathbf{\Sigma}_\phi \mid x_1 \leq x_2\}$	4221
$\mathbf{\Sigma}_{\phi\wedge\psi} = \mathbf{\Sigma}_\psi$	8160	$\mathbf{X}_{\phi\wedge\psi} = (\mathbf{X}_\phi \cap \mathbf{X}_\psi) \cup (\mathbf{X}_\psi \setminus \mathbf{\Sigma}_\phi)$	4221

Table 17: Sample and property set of the conjunction $\phi\wedge\psi$ with $\mathbf{\Sigma}_\phi \subset \mathbf{\Sigma}_\psi$ and $\mathbf{\Sigma}_\phi \neq \mathbf{\Sigma}_\psi$

Item P (Section 3.3)

The height and weight of the Dallas Mavericks and Utah Jazz players are juxtaposed in the following table and allow to count the cardinal number of $\mathbf{X}_\phi = \{(x_1,x_2) \mid x_1 \text{ taller than } x_2\}$ as 162; $\mathbf{X}_\psi = \{(x_1,x_2) \mid x_1 \text{ heavier than } x_2\}$ as 156; $\mathbf{X}_{\phi\wedge\psi} = \{(x_1,x_2) \mid x_1 \text{ taller and heavier than } x_2\}$ as 128.

Σ_1 = Dallas Mavericks						Σ_2 = Utah Jazz					
Height cm	Weight kg	Height cm	Weight kg	Height cm	Weight kg	Height cm	Weight kg	Height cm	Weight kg	Height cm	Weight kg
178	82	196	84	208	109	183	86	193	88	201	105
185	86	196	93	208	109	185	98	193	103	203	103
188	84	198	105	213	109	185	79	196	113	206	99
193	97	201	100	221	109	188	86	196	93	208	112
196	88	201	100	224	132	190	91	201	104	216	117
196	98	201	104			190	95	201	100		

Table 18: Height and weight comparison of Dallas Mavericks and Utah Jazz

Item Q (Section 3.2)
In (27), the truth degree of the conjunction of ϕ (*The Dallas Mavericks players are taller than the Denver Nuggets players*) and ψ (*The Dallas Mavericks players are even the tallest team of the NBA*) can be computed from the figures in table 14 (item N) above (ϕ) and those in table 16 (item O) above (ψ). The cardinal number of $\mathbf{X}_\phi = \{(x_1,x_2) \mid x_1 > x_2\}$ is 108 and of $\mathbf{X}_\psi = \{(x_1,x_2) \mid x_1 > x_2\}$ is 3812. The truth degree of $\phi\wedge\psi$ can be computed by following the steps indicated in table 19. It is 47% (3812/8160).

Sample:	Cardinality	Property set:	Cardinality
$\mathbf{\Sigma}_\phi$ = {Height of Dallas Mavericks} × {Height of Denver Nuggets}	272	$\mathbf{X}_\phi = \{(x_1,x_2) \in \mathbf{\Sigma}_\phi \mid x_1 > x_2\}$	108
$\mathbf{\Sigma}_\psi$ = {Height of Dallas Mavericks} × {Height of NBA without DM}	8160	$\mathbf{X}_\psi = \{(x_1,x_2) \in \mathbf{\Sigma}_\psi \mid x_1 > x_2\}$	3812
		$\mathbf{X}_\phi \cap \mathbf{X}_\psi = \{(x_1,x_2) \in \mathbf{\Sigma}_\phi \mid x_1 > x_2\}$	108
$\mathbf{\Sigma}_\psi \setminus \mathbf{\Sigma}_\phi$ = {DM heights} × {NBA heights without DM and DN}	7888	$\mathbf{X}_\psi \setminus \mathbf{\Sigma}_\phi = \{(x_1,x_2) \in \mathbf{\Sigma}_\psi \setminus \mathbf{\Sigma}_\phi \mid x_1 > x_2\}$	3704
$\mathbf{\Sigma}_{\phi\wedge\psi} = \mathbf{\Sigma}_\psi$	8160	$\mathbf{X}_{\phi\wedge\psi} = (\mathbf{X}_\phi \cap \mathbf{X}_\psi) \cup (\mathbf{X}_\psi \setminus \mathbf{\Sigma}_\phi)$	3812

Table 19: Sample and property set of the conjunction $\phi\wedge\psi$ with $\mathbf{\Sigma}_\phi \subset \mathbf{\Sigma}_\psi$ and $\mathbf{\Sigma}_\phi \neq \mathbf{\Sigma}_\psi$

Item R (Section 3.3)
The properties of the conditional and biconditional (lemma 30, relabeled as 56) are demonstrated below.

(56) *Proof*:

a. For $[\![\phi]\!] > 0 \; and \; [\![\phi\wedge\psi]\!] < [\![\phi]\!]$:

$$[\![\phi\to\psi]\!] = \frac{|X_{\phi\to\psi}|}{|\Sigma_{\phi\to\psi}|} = \frac{|X_{\phi\wedge\psi}\times\Sigma_\phi|}{|\Sigma_{\phi\wedge\psi}\times X_\phi|} = \frac{|X_{\phi\wedge\psi}|\cdot|\Sigma_\phi|}{|\Sigma_{\phi\wedge\psi}|\cdot|X_\phi|} = \frac{[\![\phi\wedge\psi]\!]}{[\![\phi]\!]};$$

For $[\![\phi]\!] > 0 \; and \; [\![\phi]\!] < [\![\phi\wedge\psi]\!]$:

$$[\![\phi\to\psi]\!] = \frac{|X_{\phi\to\psi}|}{|\Sigma_{\phi\to\psi}|} = \frac{|\Sigma_{\phi\wedge\psi}\times X_\phi|}{|X_{\phi\wedge\psi}\times\Sigma_\phi|} = \frac{|\Sigma_{\phi\wedge\psi}|\cdot|X_\phi|}{|X_{\phi\wedge\psi}|\cdot|\Sigma_\phi|} = \frac{[\![\phi]\!]}{[\![\phi\wedge\psi]\!]}$$

b. $[\![\phi\to\psi]\!] = 0$ iff $[\![\phi]\!] > 0$ *and* $[\![\phi\wedge\psi]\!] = 0$ is a direct consequence of (29a).

c. $[\![\phi\to\psi]\!] = 1$ iff $[\![\phi]\!] = 0$ *or* $[\![\phi\wedge\psi]\!] = [\![\phi]\!]$ is a direct consequence of (29a).

d. Applying the definition (29b), we have for $[\![\phi]\!] > 0, [\![\psi]\!] > 0 \; and \; [\![\psi]\!] < [\![\phi]\!]$:

$$[\![\phi\leftrightarrow\psi]\!] = \frac{|X_{\phi\leftrightarrow\psi}|}{|\Sigma_{\phi\leftrightarrow\psi}|} = \frac{|X_\psi\times\Sigma_\phi|}{|X_\phi\times\Sigma_\psi|} = \frac{|X_\psi|\cdot|\Sigma_\phi|}{|X_\phi|\cdot|\Sigma_\psi|} = \frac{[\![\psi]\!]}{[\![\phi]\!]}$$; for the subcase $0 < [\![\phi\wedge\psi]\!] \le [\![\psi]\!]$

we have: $[\![\phi \leftrightarrow \psi]\!] = \frac{[\![\psi]\!]}{[\![\phi]\!]} = \frac{[\![\psi]\!]\cdot[\![\phi \wedge \psi]\!]}{[\![\phi \wedge \psi]\!]\cdot[\![\phi]\!]} = \frac{[\![\phi \to \psi]\!]}{[\![\psi \to \phi]\!]}$; for the other subcase $[\![\psi]\!] < [\![\phi \wedge \psi]\!]$ we have: $[\![\phi \leftrightarrow \psi]\!] = \frac{[\![\psi]\!]}{[\![\phi]\!]} = \frac{[\![\psi]\!]\cdot[\![\phi \wedge \psi]\!]}{[\![\phi \wedge \psi]\!]\cdot[\![\phi]\!]} = [\![\phi \to \psi]\!]\cdot[\![\psi \to \phi]\!]$. In the same vein, we have for $[\![\phi]\!] > 0, [\![\psi]\!] > 0$ *and* $[\![\phi]\!] < [\![\psi]\!]$:

$[\![\phi \leftrightarrow \psi]\!] = \frac{|X_{\phi\leftrightarrow\psi}|}{|\Sigma_{\phi\leftrightarrow\psi}|} = \frac{|X_\phi \times \Sigma_\psi|}{|X_\psi \times \Sigma_\phi|} = \frac{|X_\phi|\cdot|\Sigma_\psi|}{|X_\psi|\cdot|\Sigma_\phi|} = \frac{[\![\phi]\!]}{[\![\psi]\!]}$; for the subcase $0 < [\![\phi \wedge \psi]\!] \le [\![\phi]\!]$ we have: $[\![\phi \leftrightarrow \psi]\!] = \frac{[\![\phi]\!]}{[\![\psi]\!]} = \frac{[\![\phi]\!]\cdot[\![\phi \wedge \psi]\!]}{[\![\phi \wedge \psi]\!]\cdot[\![\psi]\!]} = \frac{[\![\psi \to \phi]\!]}{[\![\phi \to \psi]\!]}$; for the other subcase $[\![\phi]\!] < [\![\phi \wedge \psi]\!]$ we have: $[\![\phi \leftrightarrow \psi]\!] = \frac{[\![\phi]\!]}{[\![\psi]\!]} = \frac{[\![\phi]\!]\cdot[\![\phi \wedge \psi]\!]}{[\![\phi \wedge \psi]\!]\cdot[\![\psi]\!]} = [\![\phi \to \psi]\!]\cdot[\![\psi \to \phi]\!]$.

e. This is a consequence of the definition in (29b).
f. This is a consequence of the definition in (29b).
g. This is a consequence of (28b).
h. This is a consequence of (28c).

Item S (Section 3.3)
The pair of propositions in (31)-(36) exhibit conditional and biconditional truth degrees that can be directly computed from the component truth degrees and lemma (30). The conjunctive truth degrees are retrieved from section 3.2.

Example	$[\![\phi]\!]$	$[\![\psi]\!]$	$[\![\phi \wedge \psi]\!]$	$[\![\phi \to \psi]\!]$	$[\![\psi \to \phi]\!]$	$[\![\phi \leftrightarrow \psi]\!]$
(31)	$21\% = \frac{73}{349}$	$2\% = \frac{5}{216}$	$0.5\% = \frac{5\cdot 73}{216\cdot 349}$	$2\% = \frac{5}{216}$	$21\% = \frac{73}{349}$	$11\% = \frac{5\cdot 349}{216\cdot 73}$
(32)	$21\% = \frac{73}{349}$	$19\% = \frac{80}{421}$	$17\% = \frac{107}{607}$	$84\% = \frac{107\cdot 349}{607\cdot 73}$	$93\% = \frac{107\cdot 421}{607\cdot 80}$	$91\% = \frac{349\cdot 80}{421\cdot 73}$
(33)	$82\% = \frac{9}{11}$	$21\% = \frac{73}{349}$	$71\% = \frac{68}{96}$	$87\% = \frac{68\cdot 11}{96\cdot 9}$	$30\% = \frac{73\cdot 96}{349\cdot 68}$	$26\% = \frac{73\cdot 11}{349\cdot 9}$
(34)	$56\% = \frac{162}{289}$	$40\% = \frac{108}{272}$	$48\% = \frac{270}{561}$	$86\% = \frac{270\cdot 289}{561\cdot 162}$	$83\% = \frac{561\cdot 108}{270\cdot 272}$	$71\% = \frac{289\cdot 108}{162\cdot 272}$
(35)	$56\% = \frac{162}{289}$	$54\% = \frac{156}{289}$	$44\% = \frac{128}{289}$	$79\% = \frac{128}{162}$	$82\% = \frac{128}{156}$	$96\% = \frac{156}{162}$
(36)	$40\% = \frac{108}{272}$	$47\% = \frac{3812}{8160}$	$47\% = \frac{3812}{8160}$	$74\% = \frac{172\cdot 3812}{108\cdot 8160}$	100%	$74\% = \frac{172\cdot 3812}{108\cdot 8160}$

Table 20: Truth degrees of the Probabilistic Conditional and Biconditional

Item T (Section 3.3)
The sample and property set of $\neg\phi \vee \psi$ and $(\phi \to \psi) \wedge (\psi \to \phi)$ for the examples (31)-(36), can be calculated by following the definition of conjunction and disjunction in (19) and by retrieving the data of previous tables, as indicated in table 21. (The last two lines in each block in table 21 provide the samples and property sets for each formula. The cardinality of these samples and property sets yields the truth degrees in table 2, section 3.3).

Property Set and Sample	Cardinality	Property Set and Sample	Cardinality
(31) $\mathbf{X}_\phi$	73	$\mathbf{X}_\psi$	5
$\mathbf{X}_{\neg\phi} \times \mathbf{X}_\psi$	1380		
(table 6, 7) $\mathbf{X}_\phi \times \mathbf{X}_\psi$	365	$\mathbf{X}_{\neg\phi} \times \mathbf{X}_{\neg\psi}$	58236
$\mathbf{X}_{\neg\phi\vee\psi} = (\mathbf{X}_{\neg\phi} \times \mathbf{X}_\psi) \cup (\mathbf{X}_\phi \times \mathbf{X}_\psi) \cup (\mathbf{X}_{\neg\phi} \times \mathbf{X}_{\neg\psi})$	59981	$\mathbf{X}_{(\neg\phi\vee\psi)\wedge(\neg\psi\vee\phi)} = \mathbf{X}_{\neg\phi\vee\psi} \cap \mathbf{X}_{\neg\psi\vee\phi} =$	
$\mathbf{\Sigma}_{(\neg\phi\vee\psi)\wedge(\neg\phi\vee\psi)} = \mathbf{\Sigma}_{\neg\phi\vee\psi} = \mathbf{\Sigma}_{\neg\psi\vee\phi} = \mathbf{\Sigma}_\phi \times \mathbf{\Sigma}_\psi$	75384	$= (\mathbf{X}_\phi \times \mathbf{X}_\psi) \cup (\mathbf{X}_{\neg\phi} \times \mathbf{X}_{\neg\psi})$	58601
(32) $\mathbf{X}_\phi$	73	$\mathbf{X}_\psi$	80
$\mathbf{X}_\phi \setminus \mathbf{\Sigma}_\psi$	33	$\mathbf{X}_{\neg\phi} \setminus \mathbf{\Sigma}_\psi$	173 = 206-33
(table 6, 7,12) $\mathbf{X}_\phi \cap \mathbf{X}_\psi$	31	$\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi}$	97
$\mathbf{X}_{\neg\phi\vee\psi} = (\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi}) \cup (\mathbf{X}_{\neg\phi} \setminus \mathbf{\Sigma}_\psi) \cup \mathbf{X}_\psi$	350	$\mathbf{X}_{(\neg\phi\vee\psi)\wedge(\neg\psi\vee\phi)} = \mathbf{X}_{\neg\phi\vee\psi} \cap \mathbf{X}_{\neg\psi\vee\phi} =$	
$\mathbf{\Sigma}_{(\neg\phi\vee\psi)\wedge(\neg\phi\vee\psi)} = \mathbf{\Sigma}_{\neg\phi\vee\psi} = \mathbf{\Sigma}_{\neg\psi\vee\phi} = \mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi$	627	$= (\mathbf{X}_\phi \cap \mathbf{X}_\psi) \cup (\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi})$	128
(33) $\mathbf{X}_{\phi\wedge\psi}$	68	$\mathbf{X}_{\neg\phi\wedge\psi}$	4
$\mathbf{X}_{\phi\wedge\neg\psi}$	2	$\mathbf{X}_{\neg\phi\wedge\neg\psi}$	22
(table 13) $\mathbf{X}_{\neg\phi\wedge\psi} \cup \mathbf{X}_{\phi\wedge\psi} \cup \mathbf{X}_{\neg\phi\wedge\neg\psi}$	94	$\mathbf{X}_{\neg\psi\vee\phi} = \mathbf{X}_{\phi\wedge\neg\psi} \cup \mathbf{X}_{\neg\phi\wedge\neg\psi} \cup \mathbf{X}_{\phi\wedge\psi}$	92
$\mathbf{X}_{\neg\phi\vee\psi} = \mathbf{X}_{\neg\phi\wedge\psi} \cup \mathbf{X}_{\phi\wedge\psi} \cup \mathbf{X}_{\neg\phi\wedge\neg\psi}$	94	$\mathbf{X}_{(\neg\phi\vee\psi)\wedge(\neg\psi\vee\phi)} = \mathbf{X}_{\neg\phi\vee\psi} \cap \mathbf{X}_{\neg\psi\vee\phi} =$	
$\mathbf{\Sigma}_{(\neg\phi\vee\psi)\wedge(\neg\phi\vee\psi)} = \mathbf{\Sigma}_{\neg\phi\vee\psi} = \mathbf{\Sigma}_{\neg\psi\vee\phi} = \mathbf{\Sigma}_{\phi\wedge\psi}$	96	$= \mathbf{X}_{\phi\wedge\psi} \cup \mathbf{X}_{\neg\phi\wedge\neg\psi}$	90
(34) $\mathbf{X}_\phi$	162	$\mathbf{X}_\psi$	108
$\mathbf{X}_{\neg\phi}$	127	$\mathbf{X}_{\neg\psi}$	164
(table 15) $\mathbf{X}_{\neg\phi} \cup \mathbf{X}_\psi$	235	$\mathbf{X}_\phi \cup \mathbf{X}_{\neg\psi}$	326
$\mathbf{X}_{\neg\phi\vee\psi} = \mathbf{X}_{\neg\phi} \cup \mathbf{X}_\psi$	235	$\mathbf{X}_{(\neg\phi\vee\psi)\wedge(\neg\psi\vee\phi)} = \mathbf{X}_{\neg\phi\vee\psi} \cap \mathbf{X}_{\neg\psi\vee\phi} =$	
$\mathbf{\Sigma}_{(\neg\phi\vee\psi)\wedge(\neg\phi\vee\psi)} = \mathbf{\Sigma}_{\neg\phi\vee\psi} = \mathbf{\Sigma}_{\neg\psi\vee\phi} = \mathbf{\Sigma}_\phi \cup \mathbf{\Sigma}_\psi$	561	$= (\mathbf{X}_\phi \cap \mathbf{X}_\psi) \cup (\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi})$	0
(35) $\mathbf{X}_\phi$	162	$\mathbf{X}_\psi$	156
$\mathbf{X}_{\neg\phi}$	127	$\mathbf{X}_{\neg\psi}$	133
(table 18) $\mathbf{X}_{\neg\phi} \cup \mathbf{X}_\psi$	255	$\mathbf{X}_\phi \cup \mathbf{X}_{\neg\psi}$	275
$\mathbf{X}_{\neg\phi\vee\psi} = \mathbf{X}_{\neg\phi} \cup \mathbf{X}_\psi$	255	$\mathbf{X}_{(\neg\phi\vee\psi)\wedge(\neg\psi\vee\phi)} = \mathbf{X}_{\neg\phi\vee\psi} \cap \mathbf{X}_{\neg\psi\vee\phi} =$	
$\mathbf{\Sigma}_{(\neg\phi\vee\psi)\wedge(\neg\phi\vee\psi)} = \mathbf{\Sigma}_{\neg\phi\vee\psi} = \mathbf{\Sigma}_{\neg\psi\vee\phi} = \mathbf{\Sigma}_\phi = \mathbf{\Sigma}_\psi$	289	$= (\mathbf{X}_\phi \cap \mathbf{X}_\psi) \cup (\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi})$	241
(36) $\mathbf{X}_\phi$	108	$\mathbf{X}_\psi$	3812
$\mathbf{X}_{\neg\phi}$	164	$\mathbf{X}_{\neg\psi}$	4348
(table 19) $\mathbf{X}_\phi \cap \mathbf{X}_\psi$	108	$\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi}$	164
$\mathbf{X}_{\neg\phi\vee\psi} = (\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi}) \cup \mathbf{X}_\psi$	3920	$\mathbf{X}_{(\neg\phi\vee\psi)\wedge(\neg\psi\vee\phi)} = \mathbf{X}_{\neg\phi\vee\psi} \cap \mathbf{X}_{\neg\psi\vee\phi} =$	
$\mathbf{\Sigma}_{(\neg\phi\vee\psi)\wedge(\neg\phi\vee\psi)} = \mathbf{\Sigma}_{\neg\phi\vee\psi} = \mathbf{\Sigma}_{\neg\psi\vee\phi} = \mathbf{\Sigma}_\psi$	8160	$= (\mathbf{X}_\phi \cap \mathbf{X}_\psi) \cup (\mathbf{X}_{\neg\phi} \cap \mathbf{X}_{\neg\psi})$	272

Table 21: Truth Degree Calculation of the material Conditionals/Biconditionals

Item U (Section 3.4)
The proof of Lemma (38), relabeled as (57), is provided below.

(57) *Proof*

a. We start from (i) and (ii) and assume that ϕ and ψ as well as χ and ψ are independent. It follows that the samples $\mathbf{\Sigma}_\chi$ and $\mathbf{\Sigma}_\psi$ respectively $\mathbf{\Sigma}_\phi$ and $\mathbf{\Sigma}_\psi$ are uncorrelated. From (19) we can infer that that $\mathbf{\Sigma}_{\chi\wedge\phi}$ and $\mathbf{\Sigma}_\psi$ are uncorrelated or put differently that $(\chi \wedge \phi)$ and ψ are independent. Because of this independence it is the case that $[\![\chi\wedge\phi]\!]\cdot[\![\chi\wedge\psi]\!] = [\![\chi\wedge\phi]\!]\cdot[\![\chi]\!]\cdot[\![\psi]\!] = [\![\chi\wedge\phi\wedge\psi]\!]\cdot[\![\chi]\!]$. If we divide both sides of the equation by $[\![\chi]\!]\cdot[\![\chi]\!]$, it follows from the way conditionals are defined that $[\![\chi\rightarrow(\phi\wedge\psi)]\!] = [\![\chi\rightarrow\phi]\!]\cdot[\![\chi\rightarrow\psi]\!]$ or that ϕ and ψ are conditionally independent given χ.

b. We start again from (i) and (ii). We spell out (ii): $\mathbf{\Sigma}_\chi$ and $\mathbf{\Sigma}_\psi$ are uncorrelated, as are $\mathbf{\Sigma}_\chi$ and $\mathbf{\Sigma}_\phi$. From (19) we can conclude that $\mathbf{\Sigma}_\chi$ and $\mathbf{\Sigma}_{\psi\wedge\phi}$ are uncorrelated, or put differently that χ and $(\psi \wedge \phi)$ are independent. Now $[\![\chi\rightarrow(\phi\wedge\psi)]\!] = [\![\chi\rightarrow\phi]\!]\cdot[\![\chi\rightarrow\psi]\!]$ means that $[\![\chi\wedge\phi\wedge\psi]\!]\cdot[\![\chi]\!] = [\![\chi\wedge\phi]\!]\cdot[\![\chi\wedge\psi]\!]$. From the independence of χ and $(\psi \wedge \phi)$ and from (ii), we can conclude that $[\![\phi\wedge\psi]\!] = [\![\phi]\!]\cdot[\![\psi]\!]$ or that ϕ and ψ are independent.

Item V (Section 3.4)
In examples (40), let χ (There will be a pandemic next year), ϕ (*Clorox will strongly increase sales of its disinfecting products*) and ψ (*the car rental company Hertz will file for bankruptcy*). In (41), the last sentence is exchanged by ψ (*Kimberley-Clark will greatly boost sales of its paper towels*). The truth degree of the complex conditional $\chi \rightarrow (\phi \wedge \psi)$ draws on the data of table 23. (Each time the property sets share the premises and filters of the sample and are filtered by one additional microeconomic property.)

	Premise	Macroeconomic filter	Microeconomic filter	Cardinality
$\boldsymbol{\Sigma}_{\chi\wedge\psi}$:	Hertz $\in$ [Competitors] $\cup$ [Partners]	$\wedge$ ([Event] = “1”)		59
$\mathbf{X}_{\chi\wedge\psi}$:		([Event] = “pandemic”	$\wedge$ [Bankruptcy Year+1] = “Yes”	52
$\boldsymbol{\Sigma}_{\chi\wedge\phi\wedge\psi}$:	Clorox or Hertz $\in$ [Competitors] $\cup$ [Partners]	$\wedge$ ([Event] = “1”)		145
$\mathbf{X}_{\chi\wedge\phi\wedge\psi}$:		([Event] = “pandemic”	$\wedge$ ([Sales In/Decrease Year+1] $\geq$ 9% or [Bankruptcy Year+1] = “Yes”)	113
$\boldsymbol{\Sigma}_{\chi\wedge\psi}$:	KC $\in$ [Competitors] $\cup$ [Partners]	$\wedge$ ([Event] = “1”)		114
$\mathbf{X}_{\chi\wedge\psi}$:		([Event] = “pandemic”	$\wedge$ [Sales In/Decrease Year+1] $\geq$ 9%	95
$\boldsymbol{\Sigma}_{\chi\wedge\phi\wedge\psi}$:	Clorox or KC $\in$ [Competitors] $\cup$ [Partners]	$\wedge$ ([Event] = “1”)		209
$\mathbf{X}_{\chi\wedge\phi\wedge\psi}$:		([Event] = “pandemic”	$\wedge$ [Sales In/Decrease Year+1] $\geq$ 9%	187

Table 23: Performance of Hertz and Kimberley-Clark under pandemic

Examples (39)-(41) depend on the data of tables 7, 12 and 13. In (39), the sentences χ, ϕ and ψ are mutually independent. Their conjunctive truth degrees can be calculated as multiplication.

Example	(39)	(40)	(41)
$[\![\chi]\!]$	64%	82%	82%
$[\![\phi]\!]$	21%	21%	21%
$[\![\psi]\!]$	41%	2%	19%
$[\![\phi \wedge \psi]\!]$	$9\% = \frac{73 \cdot 214}{349 \cdot 523}$	$0.5\% = \frac{73 \cdot 5}{349 \cdot 216}$	$17\% = \frac{107}{627}$
$[\![\phi]\!] \cdot [\![\psi]\!]$	$9\% = \frac{73}{349} \cdot \frac{214}{523}$	$0.5\% = \frac{73}{349} \cdot \frac{5}{216}$	$4\% = \frac{73}{349} \cdot \frac{80}{421}$
$[\![\chi \wedge \phi]\!]$	$13\% = \frac{9 \cdot 73}{14 \cdot 349}$	$71\% = \frac{68}{96}$	$71\% = \frac{68}{96}$
$[\![\chi \rightarrow \phi]\!]$	21%	$87\% = \frac{68 \cdot 11}{96 \cdot 9}$	$87\% = \frac{68 \cdot 11}{96 \cdot 9}$
$[\![\chi \wedge \psi]\!]$	$26\% = \frac{9 \cdot 214}{14 \cdot 523}$	$88\% = \frac{52}{59}$	$83\% = \frac{95}{114}$
$[\![\chi \rightarrow \psi]\!]$	41%	$72\% = \frac{52 \cdot 9}{59 \cdot 11}$	$68\% = \frac{95 \cdot 9}{114 \cdot 11}$
$[\![\chi \rightarrow \phi]\!] \cdot [\![\chi \rightarrow \psi]\!]$	$9\% = \frac{73}{349} \cdot \frac{214}{523}$	$62\% = \frac{68}{96} \cdot \frac{52}{59}$	$59\% = \frac{68}{96} \cdot \frac{95}{114}$
$[\![\chi \wedge \phi \wedge \psi]\!]$	$6\% = \frac{9 \cdot 73 \cdot 214}{14 \cdot 349 \cdot 523}$	$78\% = \frac{113}{145}$	$89\% = \frac{187}{209}$
$[\![\chi \rightarrow (\phi \wedge \psi)]\!]$	$9\% = \frac{73 \cdot 214}{349 \cdot 523}$	$95\% = \frac{113 \cdot 11}{145 \cdot 9}$	$91\% = \frac{209 \cdot 9}{187 \cdot 11}$

Table 24: Conditional (In)dependence of Examples (39)-(41)

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