

# PREDICATE-INDUCED PERMUTATION GROUPS

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## Abstract

Natural languages abound in combinatorial phenomena that are related to the predicate of the sentence and its ability to permute noun phrase arguments. After compiling several illustrative phenomena of natural languages, I propose a novel analysis in terms of *permutation groups*, a concept borrowed from mathematical combinatorics that is ubiquitous in applied sciences. I show that each natural language predicate of degree  $n$  ( $n$  natural number) can be associated with two permutation groups of degree  $n$ . The first group measures the predicate's flexibility to permute arguments in two independent events, whereas the second group captures permutations in two dependent events. These groups serve as linguistic tools to help predict the predicate's grammaticality pattern in a range of natural language constructions.

*Keywords:* Permutation group, permutation-(in)variance, natural language predicate, T×W-frames

## 1. Combinatorics in linguistics: A review

In philosophy and linguistics, combinatorial tools have been discussed previously, notably the concepts of *reflexivity*, *symmetry* and *transitivity* which are the ingredients of *equivalence relations*. Scholars were mainly interested in philosophical and cognitive accounts of the concepts of *equality*, *identity*, *similarity* etc. Quine (1969: 114-138) and Sovran (1992: 329) remarked for example that the notion of similarity notoriously resists any formal characterization as it fails to be transitive and thus to be an equivalence relation.

Several scholars also introduced the combinatorial notion of *permutation* to linguistics – in the context of generalized quantifier theory. On the following two pages, I illustrate this use of permutation and explain how it differs from the use made in this paper. Barwise & Cooper (1981) pioneered the view of noun phrases and noun determiners as quantifiers, called *generalized quantifiers*. Using type-theoretic notations (and replacing the Montague symbol “e” for “entity” by “1”), we can distinguish three types of generalized quantifiers:

- (i)  $\langle 1 \rangle$  quantifiers are full noun phrases like *John, these students, all teachers*;
- (ii)  $\langle 1, 1 \rangle$  quantifiers are one-place determiners like *all, no, most*;
- (iii)  $\langle \langle 1, 1 \rangle, 1 \rangle$  quantifiers are two-place determiners like *more...than, less...than*.

These linguistic expressions are mathematically interpreted in  $E$ , a universe of objects. For, example, the  $\langle 1 \rangle$  quantifier *John* refers to all individuals in  $E$  whose name is *John*. These individuals can be understood as a set of singletons  $\{\{a\}, \{b\}, \dots\}$ . The  $\langle 1 \rangle$  quantifier *all teachers* refers to all exhaustive groups of individuals who are teachers in a given situation. We can thus interpret *all teachers* as a set  $\{A, B, \dots\}$  of subsets of  $E$ . The  $\langle 1, 1 \rangle$  quantifier *all* refers to pairs of groups of individuals  $(A, B)$  such that all individuals of  $A$  are also individuals in  $B$ . Put differently, *all* can be interpreted as a set of pairs  $\{(A, B) \mid A \subseteq B\}$ . The  $\langle \langle 1, 1 \rangle, 1 \rangle$  quantifier *more...than* (as in *the girls are more intelligent than the boys*) can be viewed as a triple of groups of individuals  $(A, B, C)$  such that the set of individuals that are both in  $A$  (*girls*) and  $C$  (*intelligent individuals*) is larger than the set of individuals that are both in  $B$  (*boys*) and  $C$  (*intelligent individuals*). On a technical level, *more...than* can thus be understood as the set  $\{(A, B, C) \mid \text{card}(A \cap C) \geq \text{card}(B \cap C)\}$ .

To sum up, if  $\wp(E)$  denotes the powerset of  $E$ , that is the set of all subsets of  $E$ , then we can interpret each generalized quantifier in the following way.

- (i)  $\langle 1 \rangle$  quantifiers denote subsets  $Q \subseteq \wp(E)$ ;
- (ii)  $\langle 1, 1 \rangle$  quantifiers denote subsets  $Q \subseteq \wp(E) \times \wp(E)$ ;
- (iii)  $\langle \langle 1, 1 \rangle, 1 \rangle$  quantifiers denote subsets  $Q \subseteq \wp(E) \times \wp(E) \times \wp(E)$ .

Several scholars (e.g. van Benthem 1984; Keenan & Stavi 1986; Keenan & Westerstahl 1997) made use of permutations for characterizing a special property of  $\langle 1,1 \rangle$  quantifiers, called *permutation-invariance*. The linguistic idea behind *permutation-invariance* is familiar and corresponds to the intuitive notion of indefiniteness. It is in the formal apparatus of generalized quantifiers that this property has an interesting representation.  $\langle 1,1 \rangle$  quantifiers are either definite or indefinite. The meaning of definite  $\langle 1,1 \rangle$  quantifiers does not only depend on the size of the referent but also on a context and a familiarity relation. However, indefinite  $\langle 1,1 \rangle$  quantifiers, such as *all*, *any*, *a*, only depend on size properties not on the identity of the referent. Put differently, indefinite  $\langle 1,1 \rangle$  quantifiers tolerate substitutions of referents that preserve their size but change their identity. This property can be captured by the notion of permutation which has several slightly different meanings in mathematics. In the general case, a permutation is a bijective (injective and surjective) map  $\pi: E \rightarrow E$ . Indefinite quantifiers are invariant to permutations under substitution.<sup>1</sup>

**(1) Permutation-invariance:** A quantifier  $Q \subseteq \wp(E) \times \wp(E)$  is *permutation-invariant* iff for all permutations  $\pi: E \rightarrow E$  and all  $A, B \subseteq E$  we have  
 $Q\pi A \pi B$  iff  $QAB$ .

The property of permutation-invariance contrasts with the use of permutations made in this paper. The first difference, though minor, concerns the lexical class the notion of permutation is intended for. We apply permutations in this paper to verbal predicates not nominal determiners. At a technical level, however, this difference does not matter very much. Verbal predicates and nominal determiners can be interpreted in a similar way.

- (i) intransitive predicates denote subsets  $P \subseteq \wp(E)$ ;
- (ii) monotransitive predicates denote subsets  $P \subseteq \wp(E) \times \wp(E)$ ;
- (iii) ditransitive predicates denote subsets  $P \subseteq \wp(E) \times \wp(E) \times \wp(E)$ .

The notion of permutation-invariance, as defined in (1), is unproductive for monotransitive verbal predicates as almost no verbal predicate is permutation-invariant (see Westerstahl 1985: 396 for a similar comment on adverbs which he interprets as in ii).

Permutations are a productive tool for verbal predicates if we use them not to substitute groups of individuals but rather to swap argument slots. This idea would be much closer to the original sense permutations have in discrete mathematics. In discrete combinatorics, a permutation is a bijection  $\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$  of a finite (ordered) set onto itself. As I illustrate in §2, there are many linguistic phenomena that are sensitive to the degree of flexibility with which a natural language predicate can swap its arguments. For example, the three-place predicate *give* like in *John gives Mary a book* is compatible with  $\pi_1$  but incompatible with  $\pi_2$ , as defined below:

$$\begin{array}{l|l} \pi_1: \{1,2,3\} \rightarrow \{1,2,3\} & \pi_2: \{1,2,3\} \rightarrow \{1,2,3\} \\ \hline 1 \rightarrow 2 & 1 \rightarrow 3 \\ 2 \rightarrow 1 & 2 \rightarrow 2 \\ 3 \rightarrow 3 & 3 \rightarrow 1 \end{array}$$

<sup>1</sup> Stabler & Keenan (2003) applied the idea of *permutation-invariance* also to automata theory in computer science. For Minimalist Languages (a subtype of Multi-Component Context-Free Languages), they employ permutations for modeling the notion of *structural similarity* within and across natural languages. Permutations or automorphisms, as they call them, are defined with respect to the set  $\mathbf{F}$  of generating functions of a minimalist grammar  $\mathbf{G}$ . Any bijection  $h: \mathbf{L}(\mathbf{G}) \rightarrow \mathbf{L}(\mathbf{G})$  is a syntactic permutation (or automorphism), if it maps every generating function  $F \in \mathbf{F}$  onto itself:  $h(F) = F$ . Two structures  $s, t \in \mathbf{L}(\mathbf{G})$  are *similar* if there is a syntactic permutation  $h: \mathbf{L}(\mathbf{G}) \rightarrow \mathbf{L}(\mathbf{G})$  such that  $h(s) = t$ . On a restricted scale, the notion of structural similarity can also be defined for lexical extensions of a minimalist language  $\mathbf{L}(\mathbf{G})$  but not for arbitrary pairs of minimalist languages  $\mathbf{L}(\mathbf{G})$  and  $\mathbf{L}(\mathbf{G}')$ . (At least, Stabler and Keenan did not indicate a way of defining this notion in the general case.) The idea of automorphism appears to be very different from the way permutations are conceptualized in discrete mathematics and also in this paper.

An  $n$ -place predicate  $P$  (whose denotation we also write by  $P$ ) may or may not be compatible with a permutation  $\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$ . The aim of this paper is to count the number of permutations compatible with a given predicate and to enlighten linguistic phenomena through this number. To sum up, we may characterize the approach of this paper not as a problem of permutation-*invariance* but as a problem of permutation-*variance*.

**(2) Permutation-variance:** Let  $P \subseteq \wp(E)^n$  be a predicate and  $\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$  be a permutation.  $P$  is  $\pi$ -variant (or  $\pi$ -compatible) iff for all  $A_1, \dots, A_n \subseteq E$  we have

$$PA_{\pi(1), \dots, \pi(n)} \text{ iff } PA_1, \dots, A_n.$$

## 2. Small corpus of linguistic combinatorial problems

To motivate permutations in linguistics, I shall illustrate that permutations help state grammaticality properties of sentence constructions. I compile several natural language phenomena and present them in the order of the permutations for which they show sensibility: the identity permutation (§2.1), the symmetric permutation (§2.2),  $S_3$  (§2.3) and  $S_4$  (§2.4). Linguistic illustrations are drawn from several languages of the world previously reported in the linguistic (typological) literature.

### 2.1 The identity permutation

The possibility of an event to be repeated with the same constellation of arguments influences the use of grammatical aspect in many languages, especially quantificational aspect. As a grammatical category, quantificational aspect (with the *experiential* and *habitual* aspects as the two major exponents)<sup>2</sup> is attested in several language families worldwide. Most languages of East Asia exhibit experiential aspect particles that are ungrammatical with sentence predicates whose referring event cannot be repeated with the same (referring) NP arguments. This selectional restriction is well known by specialists of East Asian languages as the “repeatability property” (for Chinese see Pan & Lee 2004, for Japanese see Inoue 1975, for Korean see Kim 1998, for the Yi languages see Gerner 2004). The repeatable constraint also applies to the habitual aspect. Furthermore, so-called weak-repeatable (stage-level)<sup>3</sup> predicates are compatible with the experiential and habitual aspects, whereas strong-repeatable (individual-level)<sup>4</sup> predicates are incompatible (Gerner 2004:1347).

We employ the terms *unrepeatable*, *weak-repeatable* and *strong-repeatable* defined later more rigorously. These repeatability properties and their selectional restrictions on quantificational aspect are shown for the two sentence-end particles  $ta^{33}$  (experiential) and  $k^w en^{53}$  (habitual) in Kam.<sup>5</sup>

- Kam language* (Kam-Tai family: Guizhou Province, P.R. of China)
- |   |  |  |                |
|---|--|--|----------------|
| (3) a.  | *mao <sup>33</sup> tɔi <sup>55</sup> ta <sup>33</sup> / k <sup>w</sup> en <sup>53</sup> .<br>3P SG    die    EXP    HAB  |  | P unrepeatable |
| ‘*He experienced dying. / He used to die.’                                    |  |  |                |
| b.  | *mao <sup>33</sup> kao <sup>53</sup> mɔi <sup>31</sup> kuk <sup>323</sup> tɔ <sup>33</sup> ta <sup>33</sup> / k <sup>w</sup> en <sup>53</sup> .<br>3P SG    wear out CL    clothes DEM:DIST EXP    HAB |  | P unrepeatable |
| ‘*He experienced wearing out the clothes. / He used to wear out the clothes.’ |  |  |                |

<sup>2</sup> The grammatical category of experiential aspect is restricted to two major regions of the world: Africa and East Asia (Dahl 1985: 140). The languages in East Asia in which theoretical studies for the *experiential* aspect were proposed include Korean, Japanese, Chinese, and some Tibeto-Burman languages. The habitual aspect is attested in all major language families of the world (cf. Bybee et al. 1994) and surfaces either as inflectional category in conjugation systems (e.g. French past tense conjugation) or as sentence-end particle after the verb (e.g. Kam).

<sup>3</sup> For the notions of stage-level and individual-level, see Kratzer (1995) and also Carlson (1977).

<sup>4</sup> See previous footnote.

<sup>5</sup> The numbers <sup>55</sup>, <sup>13</sup> etc. are tone markers and indicate relative pitch on a scale from 1 (lowest) to 5 (highest). The first number represents the beginning and the second number the end of the tonal contour. The transcription of sounds in this paper follows the International Phonetic Alphabet without shortcuts. For the interlinear abbreviations used in the examples, refer to the section of abbreviations.

- |        |   |   |
|--------|---|---|
| (4) a. | mao <sup>33</sup> ko <sup>55</sup> tɕa <sup>31</sup> tɕa <sup>31</sup> ta <sup>33</sup> / k <sup>w</sup> en <sup>53</sup> .<br>3P SG laugh IDE-IDE EXP HAB                        | P weak-repeatable<br>(P stage-level)        |
|        | ‘He experienced laughing with a roaring voice. / He used to laugh with a roaring voice.’  |   |
| b.     | mao <sup>33</sup> laŋ <sup>13</sup> to <sup>55</sup> mak <sup>323</sup> ta <sup>33</sup> / k <sup>w</sup> en <sup>53</sup> .<br>3P SG open door, gate big EXP HAB                 | P weak-repeatable<br>(P stage-level)        |
|        | ‘He experienced opening the big gate. / He used to open the big gate.’  |   |
| (5) a. | *tɕak <sup>11</sup> tɕən <sup>11</sup> ta <sup>33</sup> p <sup>h</sup> aŋ <sup>35</sup> ta <sup>33</sup> / k <sup>w</sup> en <sup>53</sup> .<br>CL mountain DEM:DIST high EXP HAB | P strong-repeatable<br>(P individual-level) |
|        | ‘*The mountain was once high. / The mountain used to be high.’  |   |
| b.     | *jao <sup>11</sup> lao <sup>31</sup> mao <sup>33</sup> ta <sup>33</sup> / k <sup>w</sup> en <sup>53</sup> .<br>1P SG old(er) 3P SG EXP HAB  | P strong-repeatable<br>(P individual-level) |
|        | ‘*I experienced being older than him. / I used to be older than him.’   |   |

The possibility of repeating an event with the same referring arguments is a combinatorial property of the predicate, a property related to the identity permutation.

## 1.2 The symmetric permutation

The ability of a predicate to swap arguments in two independent or two dependent events interacts with several sentence constructions.

First, many native languages of North America involve inverse marking which was mistakenly viewed as a sort of passive marking (Whaley 1997). Inverse marking encodes a subject/object reversal and is expressed by a verbal affix to indicate that the arguments are swapped in comparison to a related construction, called the *direct construction*, in which the affix is missing. The availability of inverse marking depends on the symmetry type of the predicate. Data originate from Kutenai (Dryer 1994, 1996, 2008), a language isolate spoken in British Columbia (Canada). In Kutenai, inverse marking is possible, if the predicate allows the symmetric permutation of its arguments, as in (4a+b), but is impossible if it is basically asymmetric, as in (5a+b). The verbal inverse affix is *-aps-*.

*Kutenai language* (language isolate assimilated with Algonquian family: Canada, USA)

- |        |  |                                |
|--------|--|--------------------------------|
| (6) a. | wu kat-i ni?-s paŋkiy-s ni? titqat'<br>see-INDIC the-OBV woman-OBV the man                           | P symmetric<br>Direct clause   |
|        | ‘The man saw the woman.’   |                                |
| b.     | wu kat-aps-i ni?-s paŋkiy-s ni? titqat'<br>see-INV-INDIC the-OBV woman-OBV the man                   | P symmetric<br>Inverse clause  |
|        | ‘The woman saw the man.’   |                                |
| (7) a. | taxa-s n=ik-ni skinkuɕ ni?-s ?akuʎal-s.<br>then-OBV INDIC=eat-INDIC coyote the-OBV meat-OBV          | P asymmetric<br>Direct clause  |
|        | ‘Then Coyote ate the meat.’  |                                |
| b.     | *taxa-s n=ik-aps-ni skinkuɕ ni?-s ?akuʎal-s.<br>then-OBV INDIC=eat-INV-INDIC coyote the-OBV meat-OBV | P asymmetric<br>Inverse clause |
|        | ‘*Then the meat ate Coyote.’   |                                |

Second, reciprocal constructions (with a reciprocal anaphor or a reciprocal verb affix) are sensitive to the symmetric type of the predicate too. We call these symmetric types informally *asymmetric*, *weak-symmetric* and *strong-symmetric*. Examples (8)-(10) show them for the Chinese verb affix *bǐcǐ* (‘each other’) in reciprocal constructions and in sentences in which the event with permuted arguments is negated.

*Standard Chinese* (Sinitic group: P.R. of China)

- (8) \*他们 彼此 埋葬。  
tā mén bǐcǐ mái zàng.  
3P PL RECL bury  
‘They bury each other.’ P asymmetric
- (9) a. 他们 彼此 安慰。  
tā mén bǐcǐ ān wèi.  
3P PL RECL comfort  
‘They comfort each other.’ P weak-symmetric
- b. 他 安慰 你,  
tā ān wèi nǐ  
3P SG fear 2P SG  
但 你 不 安慰 他。  
dàn nǐ bù ān wèi tā.  
but 2P SG NEG fear 3P SG  
‘He comforts you, but you do not comfort him.’ P weak-symmetric
- (10) a. 他们 彼此 相象。  
tā mén bǐcǐ xiāng xiàng.  
3P PL RECL resemble  
‘They resemble each other.’ P strong-symmetric
- b. \*他 象 你,  
tā xiàng nǐ  
3P SG resemble 2P SG  
但 你 不 象 他。  
dàn nǐ bù xiàng tā  
but 2P SG NEG resemble 3P SG  
‘He resembles you, but you do not resemble him.’ P strong-symmetric

### 1.3 The permutation group S<sub>3</sub>

In most languages, there are ditransitive predicates that can take three human NPs as arguments allowing them to be permuted in any possible way. However, only in so-called *free word order languages* do these ditransitive predicates interact with the syntactic marking system.

Free word order languages generally compensate for their syntactic flexibility with case or agreement marking. The isolating Lolo language is verb-final with free word order of its arguments. It exhibits a differential-object case marking driven by ambiguity (Gerner 2008). The morpheme *thie*<sup>21</sup> is a combined focus and case marker. It functions as a focus marker when the predicational frame of the sentence is not inherently ambiguous such as in *Mary washes three trousers*, but assumes the meaning of case marker when the predicational frame is ambiguous like in *Mary bites John*.

*Lolo language* (Tibeto-Burman family: Yunnan Province, P.R. of China)

- (11)  $\text{b}\text{ɔ}^{33}\text{l}\text{u}^{21}$   $\text{m}\text{ɔ}^{33}\text{l}\text{u}^{33}$   $\text{s}\text{ɔ}^{33}$   $\text{k}^{\text{h}}\text{ɛ}^{33}$   $\text{t}^{\text{h}}\text{i}\text{e}^{21}$   $\text{t}\text{s}^{\text{h}}\text{i}^{21}$   $\text{ɔ}^{33}$ . | *thie*<sup>21</sup> focus marker  
 name of man trousers NUM:3 CL O-marker wash DP  
 S O V

‘Bolu has washed THREE pairs of trousers [not just TWO]’

- (12) a.  $\frac{si^{33}ka^{55}}{\text{tree}} \quad \frac{\chi e^{33}k^h u^{33} \quad t^h i e^{21}}{\text{house} \quad \text{O-marker}} \quad \frac{ti^{55} \quad na^{33}}{\text{smash} \quad \text{broken}}$  |  $thie^{21}$  case marker  
           S                          O                          V  
 ‘The tree smashed the house.’
- b.  $\frac{si^{33}ka^{55}}{\text{tree}} \quad \frac{t^h i e^{21}}{\text{O-marker}} \quad \frac{\chi e^{33}k^h u^{33}}{\text{house}} \quad \frac{ti^{55} \quad na^{33}}{\text{smash} \quad \text{broken}}$  |  $thie^{21}$  case marker  
           O                          S                          V  
 ‘The house smashed the tree.’

The morpheme  $thie^{21}$  is the only available case marker for a wide range of semantic roles. It marks the direct object (O), the beneficiary (B) and a few other oblique semantic roles. It is used when an ambiguity between arguments arises and it is omitted when the argument roles are assigned by the predicate unambiguously. Several ditransitive predicates in Lolo can take three human arguments as in (13a). Since semantic roles are not assigned by word order, the degree of ambiguity is high if the particle  $thie^{21}$  is not used. In (13a), there are  $3! = 6$  possible interpretations.

- (13) a. (\*)  $\frac{\varrho^{21}mo^{33}}{\text{mother}} \quad \frac{\varrho^{55}no^{33}sə^{33}}{\text{name of man}} \quad \frac{bo^{33}lu^{21}}{\text{name of man}} \quad \frac{dzi^{33}}{\text{hand over}} \quad \frac{gə^{21}}{\text{PRED:give}}$   
           S/O/B                  O/S/B                  B/O/S                  V
- (i) ‘Mom handed Onose over to Bolu.’ (ii) ‘Mom handed Bolu over to Onose.’ (iii) ‘Bolu handed Mom over to Onose.’ (iv) ‘Bolu handed Onose over to Mom.’ (v) ‘Onose handed Mom over to Bolu.’ (vi) ‘Onose handed Bolu over to Mom.’

This ambiguity can be resolved by a double use of the case particle  $thie^{21}$ . The double occurrence of  $thie^{21}$  creates in turn a new ambiguity which is settled through word order. The first NP marked by  $thie^{21}$  is the direct object and the second the beneficiary. In (13b) only the relative order of O and B is fixed. The S may freely occur in any word order slot as far as the relative order of O and B is respected.

- b.  $\frac{\varrho^{55}no^{33}sə^{33}}{\text{name of man}} \quad \frac{t^h i e^{21}}{\text{O-marker}} \quad \frac{\varrho^{21}mo^{33}}{\text{mother}} \quad \frac{bo^{33}lu^{21}}{\text{name of man}} \quad \frac{t^h i e^{21}}{\text{B-marker}} \quad \frac{dzi^{33}}{\text{hand over}} \quad \frac{gə^{21}}{\text{PRED:give}}$   
           O                          S                          B
- ‘Mom handed Onose over to Bolu.’

In these examples, the combinatorial properties of the predicate are closely related to the number of occurrences of the case marker  $thie^{21}$ . We will revisit the Lolo data in §6.4.

#### 2.4 The permutation group $S_4$

Valence is the linguistic term to refer to the number of core arguments a natural language predicate takes. Most languages involve morphological strategies (e.g. affixation) to permit changes of the basic valence of a predicate. *Applicative* and *causative* are the most common morphological strategies in languages of the world to increase the valence of a predicate (Whaley 1997). In languages in which the applicative or the causative are productive morphological processes, regular ditransitive predicates can be extended into ‘quadransitive’ (4-place) predicates. If in addition the language has free word order, then these quadransitive predicates interact with the syntactic marking system.

Lolo (see §2.3, Gerner 2008) involves a productive causative suffix (the morpheme  $no^{55}$ ) that increases the valence of each predicate. For example, it transforms ditransitive predicates into quadransitive predicates by adding the argument of *causer*. If the case marker  $thie^{21}$  was not used, there would be  $4! = 24$  possible interpretations. However, Lolo curbs this extreme ambiguity by imposing the relative order CAUSER-CAUSEE so that there are only  $24/2 = 12$  possible interpretations.

Lolo language (Tibeto-Burman family: Yunnan Province, P.R. of China)

(14) a. * $\text{ɔ}^{21}\text{mo}^{33}$	$\text{na}^{55}\text{d}^{\text{v}}\text{u}^{33}$	$\text{ɔ}^{55}\text{no}^{33}\text{sə}^{33}$
mother	name of woman	name of man
CAUSER/O/B	CAUSER/CAUSEE/O/B	CAUSER/ CAUSEE/O/B
$\text{bo}^{33}\text{lu}^{21}$	<b>dzi</b> <sup>33</sup>	<b>gə</b> <sup>21</sup>
name of man	hand over	PRED:give
CAUSEE/O/B	V	PRED:cause

(i) ‘Mother made Nadu hand Onose over to Bolu.’ (ii) ‘Mother made Nadu hand Bolu over to Onose.’ (iii) ‘Mother made Onose hand Nadu over to Bolu.’ (iv) ‘Mother made Bolu hand Nadu over to Onose.’ (v) ‘Mother made Onose hand Bolu over to Nadu.’ (vi) ‘Mother made Bolu hand Onose over to Nadu.’ (vii) ‘Nadu made Onose hand mother over to Bolu.’ (viii) ‘Nadu made Bolu hand mother over to Onose.’ (ix) ‘Onose made Bolu hand mother over to Nadu.’ (x) ‘Nadu made Onose hand Bolu over to mother.’ (xi) ‘Nadu made Bolu hand Onose over to mother.’ (xii) ‘Onose made Bolu hand Nadu over to mother.’

Native Lolo would not use (14a) in communication due to its extreme ambiguity, but would postpose the case suffix  $\text{t}^{\text{h}}\text{ie}^{21}$  after the second, third and fourth NP. The resulting new ambiguity is then resolved through word order. The second NP is the CAUSEE, the third the direct object (O) and the fourth the beneficiary (B). The first NP which is not suffixed by  $\text{t}^{\text{h}}\text{ie}^{21}$  is the CAUSER.

b. $\text{ɔ}^{21}\text{mo}^{33}$	$\text{na}^{55}\text{d}^{\text{v}}\text{u}^{33}$	$\text{t}^{\text{h}}\text{ie}^{21}$	$\text{ɔ}^{55}\text{no}^{33}\text{sə}^{33}$	$\text{t}^{\text{h}}\text{ie}^{21}$
mother	name of woman	CAUSEE-marker	name of man	O-marker
CAUSER	CAUSEE	O		
$\text{bo}^{33}\text{lu}^{21}$	$\text{t}^{\text{h}}\text{ie}^{21}$	<b>dzi</b> <sup>33</sup>	<b>gə</b> <sup>21</sup>	<b>no</b> <sup>55</sup> .
name of man	B-marker	hand over	PRED:give	PRED:cause
B	V			

‘Mom made Nadu hand Onose over to Bolu.’

The permutation properties of the predicate influence thus the use of the case suffix  $\text{t}^{\text{h}}\text{ie}^{21}$  and the syntactic marking system as a whole. In §6.4, these data will be characterized with permutation groups.

### 3. The insufficiency of mereological approaches

In their survey monograph, Levin & Hovav (2005) review theories of *argument realization* and mention the mereological approach (e.g. Bach 1986; Krifka 1989, 1992; Vendler 1967) as a model of event classification by means of the inclusion properties that different subevents of a given event satisfy. The most famous classification proposed is that of the Vendlerian classes *states*, *activities*, *accomplishment* and *achievement*. Krifka (1989; 1992) replaced the last two terms by *quantized* and *bounded* activity.

Although the mereological approach is successful in explaining several grammatical phenomena (notably the *progressive* and *perfective* aspects), it cannot account for the combinatorial properties sketched in the previous section. The reason for this failure is relatively straightforward. None of the morphemes and particles presented in §2 manifests its selectional restrictions in terms of *Aktionsarten*. To show this point, I shall restrict myself to the experiential and habitual aspect. Examples in (15), taken from the Kam language, illustrate that both aspects are compatible with states, atelic activities, quantized activities and bounded activities.

(15) a.	State
	$\text{mao}^{33}$ $\text{sin}^{55}$ $\text{ta}^{33}$ / $\text{k}^{\text{w}}\text{en}^{53}$ .
	3P SG clean EXP HAB
	‘He experienced being clean. / He used to be clean.’

- b. *Atelic activity*  
 mao<sup>33</sup> l'ak<sup>11</sup> ho<sup>453</sup> ta<sup>33</sup> / k<sup>w</sup>en<sup>53</sup>.  
 3P SG steal thing, good EXP HAB  
 'He experienced stealing something. / He used to steal.'
- c. *Quantized activity*  
 mao<sup>33</sup> i<sup>55</sup> mən<sup>55</sup> tɕi<sup>55</sup> si<sup>453</sup> tən<sup>53</sup> əu<sup>31</sup> ta<sup>33</sup> / k<sup>w</sup>en<sup>53</sup>.  
 3P SG NUM:1 day eat NUM:4 CL:meal rice, food EXP HAB  
 'He experienced eating four meals in one day. / He used to eat four meals in one day.'
- d. *Bounded activity*  
 mao<sup>33</sup> səm<sup>33</sup> dəm<sup>55</sup> jao<sup>11</sup> ta<sup>33</sup> / k<sup>w</sup>en<sup>53</sup>.  
 3P SG search notice 1P SG EXP HAB  
 'He experienced finding me. / He used to find me.'

The selectional restrictions of the experiential and habitual aspects are not related to the mereological nature of Aktionsarten but to combinatorial notions, which are properties of sets of events not of single events. The theory that I develop is built on the mathematical notion of argument-permutation (Merris 2003).

In §4, I demonstrate how this idea works for intransitive, monotransitive and ditransitive predicates. For each predicate type, we sketch its combinatorial properties in a *Modal-Tense Predicate Logic* with two intensional operators:  $\Box$  (necessity) and  $\Diamond$  (possibility). The formal language will be introduced in detail in §5.1. The notion of predicate-induced permutation group is defined in §5.2 and §5.3. In §6, we provide an account of the phenomena sketched in §2 based on permutation groups.

#### 4. The notion of permutations induced by natural language predicates

Three points need to be clarified. First, the linguistic entities that a predicate can permute are semantic roles (e.g. agent, patient, recipient), syntactic roles (e.g. subject, object) and pragmatic roles (e.g. topic, comment). The theory proposed in this paper targets semantic roles; we hence only consider pairs of sentences like in (16a-b), not those in (17a-b) or (18a-b).

- (16) Permutation of the thematic roles of agent and recipient  
 a. John gave Mary a book.  
 b. Mary gave John a book.
- (17) Permutation of the syntactic roles of subject and (oblique) object  
 a. John blamed Mary.  
 b. Mary was blamed by John.
- (18) Permutation of the pragmatic roles of topic and comment  
 a. (A: What about John?) B: As for John, he listened to Mary's concert.  
 b. (A: What about Mary?) B: Mary gave a concert on the occasion of John's birthday.

Second, depending on the type of arguments it takes, the same predicate may exhibit different permutation properties. The verb *beat*, for instance, may take human and non-human NPs as direct object. For human NPs (e.g. *John beats Bill*), *beat* may permute its arguments (*Bill beats John*), whereas swapping a human subject and an inanimate object (*John beats the carpet*, *\*the carpet beats John*) is illicit. It does not ensue, however, that we must distinguish *beat* as two lexemes in the lexicon, but that the permutation properties associate with two different subcategorization frames listed under the lexeme *beat*.

Third, the availability of permutations hinges on the relatedness of the two referring events. The events *Bill buries John* and *John buries Bill* are both conceivable as two independent events, but cannot



occur as two events that grow out of each other. We therefore distinguish permutations of arguments in dependent events and in *independent* events both informing us about logical properties of the predicate.

#### 4.1 Intransitive Predicates

For intransitive predicates, there is only one permutation (the function that maps the sole argument onto itself). It generates the following *repeatability properties*. The modal operators  $\Box$  and  $\Diamond$  stand for universal and existential quantifiers of *scenarios*, an acronym of *time-world* pairs. (The scope of the operators  $\Box$  and  $\Diamond$  is indicated by square brackets.)

(19)	a.	$\mu$ -ambiguous (AMBI- $\mu$ ): $\forall x [\Diamond P(x) \wedge \Diamond P(x)]$	$\mu: \{x\} \rightarrow \{x\}$
	b.	Unrepeatable (NON- $\mu$ ): $\forall x \Box [P(x) \rightarrow \Box \neg P(x)]$	$x \rightarrow x$
	c.	Weak-repeatable (WEAK- $\mu$ ): $\forall x \Box [P(x) \rightarrow \Diamond P(x) \wedge \Diamond \neg P(x)]$	
	d.	Strong-repeatable (STRONG- $\mu$ ): $\forall x \Box [P(x) \rightarrow \Box P(x)]$	

For intransitive predicates, the property AMBI- $\mu$  in (19a) is a tautology, whereas NON- $\mu$ , WEAK- $\mu$  and STRONG- $\mu$  in (19b-d) are non-trivial. As an illustration, the predicate ‘sleep’ in (20) is WEAK- $\mu$  (weak-repeatable). If someone has slept for two days in one scenario (at one given time  $t$  and in one given world  $w$ ), then this scenario may evolve in at least two ways: it will develop into another scenario in which the person sleeps again for two days, or it will morph into a scenario in which s/he does not.

<i>Kam language</i> (Kam-Tai family: Guizhou Province, P.R. of China)		
(20)	$\text{mao}^{33}$ $\text{nak}^{55}$ $\text{ja}^{11}$ $\text{mɛn}^{55}$ $\text{ta}^{33}$ / $\text{k}^{\text{w}}\text{en}^{53}$ . 3P SG sleep NUM:2 day EXP HAB	$P$ weak-repeatable ( $P$ stage-level)
	‘He experienced sleeping for two days. / He used to sleep for two days.’	

The notions of weak/strong-repeatable are reminiscent of the concepts of stage-level and individual-level developed by Kratzer (1995; see also Carlson 1977). Kratzer developed this distinction primarily as a temporal (Davidsonian event) notion, whereas in my view the modal component is the crucial feature of predicates to theorize upon. Four permutation sets can be associated with a given  $P$ .

#### (21) Definition (Permutation sets of intransitive predicates):

- a.  $S_{ambi}(P) = \{\mu: \{x\} \rightarrow \{x\} \mid \mu \text{ bijective, AMBI-}\mu(P)\};$
- b.  $S_{non}(P) = \{\mu: \{x\} \rightarrow \{x\} \mid \mu \text{ bijective, NON-}\mu(P)\};$
- c.  $S_{weak}(P) = \{\mu: \{x\} \rightarrow \{x\} \mid \mu \text{ bijective, WEAK-}\mu(P)\};$
- d.  $S_{strong}(P) = \{\mu: \{x\} \rightarrow \{x\} \mid \mu \text{ bijective, STRONG-}\mu(P)\}.$

(22) **Examples:** a. For  $P$ :  $\text{tɔi}^{55}$  ‘die’ in the Kam language, we have

- $S_{non}(P) = \{\mu: \{x\} \rightarrow \{x\}\}$
- $S_{weak}(P) = \emptyset$
- $S_{strong}(P) = \emptyset$

b. For  $P$ :  $\text{ko}^{55}$  ‘laugh’ in the Kam language, we have

- $S_{non}(P) = \emptyset$
- $S_{weak}(P) = \{\mu: \{x\} \rightarrow \{x\}\}$
- $S_{strong}(P) = \emptyset$

c. For  $P$ :  $\text{p}^{\text{h}}\text{aŋ}^{35}$  ‘high’ in the Kam language, we have

- $S_{non}(P) = \emptyset$
- $S_{weak}(P) = \emptyset$
- $S_{strong}(P) = \{\mu: \{x\} \rightarrow \{x\}\}.$

## 4.2 Monotransitive Predicates

A monotransitive predicate  $P$  is sensitive to two permutations, the *identity* permutation and the *symmetry* permutation, written as bijections  $\tau: \{x,y\} \rightarrow \{x,y\}$  (where  $x$  and  $y$  are arbitrary arguments). For each permutation  $\tau$ , we introduce two types of measures that express  $P$ 's ability to permute arguments: one in two unrelated events, the other in two consecutive events. The first property is abbreviated as AMBI- $\tau$  whereas the second property is patterned through NON- $\tau$ , WEAK- $\tau$ , STRONG- $\tau$ .

(23) a.	$\tau_1$ -ambiguous (AMBI- $\tau_1$ ):	$\forall x,y [\Diamond P(x,y) \wedge \Diamond P(x,y)]$	$\tau_1: \{x,y\} \rightarrow \{x,y\}$
b.	Unrepeatable (NON- $\tau_1$ ):	$\forall x,y \Box [P(x,y) \rightarrow \Box \neg P(x,y)]$	
c.	Weak-repeatable (WEAK- $\tau_1$ ):	$\forall x,y \Box [P(x,y) \rightarrow \Diamond P(x,y) \wedge \Diamond \neg P(x,y)]$	
d.	Strong-repeatable (STRONG- $\tau_1$ ):	$\forall x,y \Box [P(x,y) \rightarrow \Box P(x,y)]$	
(24) a.	$\tau_2$ -ambiguous (AMBI- $\tau_2$ ):	$\forall x,y [\Diamond P(x,y) \wedge \Diamond P(y,x)]$	$\tau_2: \{x,y\} \rightarrow \{x,y\}$
b.	Asymmetric (NON- $\tau_2$ ):	$\forall x,y \Box [P(x,y) \rightarrow \Box \neg P(y,x)]$	
c.	Weak-symmetric (WEAK- $\tau_2$ ):	$\forall x,y \Box [P(x,y) \rightarrow \Diamond P(y,x) \wedge \Diamond \neg P(y,x)]$	
d.	Strong-symmetric (STRONG- $\tau_2$ ):	$\forall x,y \Box [P(x,y) \rightarrow \Box P(y,x)]$	

As illustrated in §2.1, repeatability properties related to the permutation  $\tau_1$  interact with the experiential (Kam: ta<sup>33</sup>) and habitual aspects (Kam: k<sup>w</sup>en<sup>53</sup>). Both aspects match with weak-repeatable predicates, but are incompatible with unrepeatable and strong-repeatable predicates. Moreover, as shown in §2.2, reciprocal constructions manifest selectional restrictions captured by the symmetry properties of the predicate (associated with the permutation  $\tau_2$ ). Reciprocal anaphors are compatible with weak- and strong-symmetric predicates, but illicit with asymmetric predicates.

In analogy to (21), we can define four permutation sets for each monotransitive predicate  $P$  and each pair of NP arguments  $x$  and  $y$ . The set  $S_{ambi}(P)$  can be viewed as a measure of licit permutations realized in *unconnected* events, while  $S_{non}(P)$ ,  $S_{weak}(P)$  and  $S_{strong}(P)$  gauge permutations realized in *successive* events. A permutation  $\tau: \{x,y\} \rightarrow \{x,y\}$  will be in exactly one of the three permutation sets  $S_{non}(P)$ ,  $S_{weak}(P)$  and  $S_{strong}(P)$ .

### (25) Definition (Permutation sets of monotransitive predicates):

- $S_{ambi}(P) = \{\tau: \{x,y\} \rightarrow \{x,y\} \mid \tau \text{ bijective, AMBI-}\pi(P)\};$
- $S_{non}(P) = \{\tau: \{x,y\} \rightarrow \{x,y\} \mid \tau \text{ bijective, NON-}\pi(P)\};$
- $S_{weak}(P) = \{\tau: \{x,y\} \rightarrow \{x,y\} \mid \tau \text{ bijective, WEAK-}\pi(P)\};$
- $S_{strong}(P) = \{\tau: \{x,y\} \rightarrow \{x,y\} \mid \tau \text{ bijective, STRONG-}\pi(P)\}.$

(26) Examples: a. For  $P$ : mok<sup>55</sup> ‘bury’ in the Kam language, we have

- $S_{ambi}(P) = \{\tau_1, \tau_2\}$
- $S_{non}(P) = \{\tau_1, \tau_2\}$
- $S_{weak}(P) = \emptyset$
- $S_{strong}(P) = \emptyset$

b. For  $P$ : laŋ<sup>13</sup> ‘open’ in the Kam language, we have

- $S_{ambi}(P) = \{\tau_1\}$
- $S_{non}(P) = \{\tau_2\}$
- $S_{weak}(P) = \{\tau_1\}$
- $S_{strong}(P) = \emptyset$

- c. For  $P$ : jao<sup>13</sup> ‘fear’ in the Kam language, we have
- $S_{ambi}(P) = \{\tau_1, \tau_2\}$
  - $S_{non}(P) = \emptyset$
  - $S_{weak}(P) = \{\tau_1, \tau_2\}$
  - $S_{strong}(P) = \emptyset$
- d. For  $P$ : lao<sup>31</sup> ‘old(er)’ in the Kam language, we have
- $S_{ambi}(P) = \{\tau_1, \tau_2\}$
  - $S_{non}(P) = \{\tau_2\}$
  - $S_{weak}(P) = \emptyset$
  - $S_{strong}(P) = \{\tau_1\}$
- e. For  $P$ : ɕoŋ<sup>323</sup> ‘resemble (in permanency reading)’ in the Kam language, we have
- $S_{ambi}(P) = \{\tau_1, \tau_2\}$
  - $S_{non}(P) = \emptyset$
  - $S_{weak}(P) = \emptyset$
  - $S_{strong}(P) = \{\tau_1, \tau_2\}$ .

### 4.3 Ditransitive Predicates

For three arguments  $x, y, z$ , there are exactly six possible permutations. Each permutation generates again four properties that a given ditransitive predicate  $P$  either satisfies or rejects.

(27) a.	AMBI- $\pi_1$ -compatible: $\forall x,y,z [\Diamond P(x, y, z) \wedge \Diamond P(x, y, z)]$	$\pi_1: \{x,y,z\} \rightarrow \{x,y,z\}$ $x \rightarrow x$ $y \rightarrow y$ $z \rightarrow z$
b.	NON- $\pi_1$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box \neg P(x, y, z)]]$	
c.	WEAK- $\pi_1$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Diamond P(x, y, z) \wedge \Diamond \neg P(x, y, z)]]$	
d.	STRONG- $\pi_1$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box P(x, y, z)]]$	
(28) a.	AMBI- $\pi_2$ -compatible: $\forall x,y,z [\Diamond P(x, y, z) \wedge \Diamond P(x, z, y)]$	$\pi_2: \{x,y,z\} \rightarrow \{x,y,z\}$ $x \rightarrow x$ $y \rightarrow z$ $z \rightarrow y$
b.	NON- $\pi_2$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box \neg P(x, z, y)]]$	
c.	WEAK- $\pi_2$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Diamond P(x, z, y) \wedge \Diamond \neg P(x, z, y)]]$	
d.	STRONG- $\pi_2$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box P(x, z, y)]]$	
(29) a.	AMBI- $\pi_3$ -compatible: $\forall x,y,z [\Diamond P(x, y, z) \wedge \Diamond P(y, x, z)]$	$\pi_3: \{x,y,z\} \rightarrow \{x,y,z\}$ $x \rightarrow y$ $y \rightarrow x$ $z \rightarrow z$
b.	NON- $\pi_3$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box \neg P(y, x, z)]]$	
c.	WEAK- $\pi_3$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Diamond P(y, x, z) \wedge \Diamond \neg P(y, x, z)]]$	
d.	STRONG- $\pi_3$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box P(y, x, z)]]$	
(30) a.	AMBI- $\pi_4$ -compatible: $\forall x,y,z [\Diamond P(x, y, z) \wedge \Diamond P(y, z, x)]$	$\pi_4: \{x,y,z\} \rightarrow \{x,y,z\}$ $x \rightarrow y$ $y \rightarrow z$ $z \rightarrow x$
b.	NON- $\pi_4$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box \neg P(y, z, x)]]$	
c.	WEAK- $\pi_4$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Diamond P(y, z, x) \wedge \Diamond \neg P(y, z, x)]]$	
d.	STRONG- $\pi_4$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box P(y, z, x)]]$	
(31) a.	AMBI- $\pi_5$ -compatible: $\forall x,y,z [\Diamond P(x, y, z) \wedge \Diamond P(z, x, y)]$	$\pi_5: \{x,y,z\} \rightarrow \{x,y,z\}$ $x \rightarrow z$ $y \rightarrow x$ $z \rightarrow y$
b.	NON- $\pi_5$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box \neg P(z, x, y)]]$	
c.	WEAK- $\pi_5$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Diamond P(z, x, y) \wedge \Diamond \neg P(z, x, y)]]$	
d.	STRONG- $\pi_5$ -compatible: $\forall x,y,z [\Box [P(x, y, z) \rightarrow \Box P(z, x, y)]]$	

(32) a.	AMBI- $\pi_6$ -compatible:	$\forall x,y,z [\Diamond P(x,y,z) \wedge \Diamond P(z,y,x)]$	$\pi_6: \{x,y,z\} \rightarrow \{x,y,z\}$
b.	NON- $\pi_6$ -compatible:	$\forall x,y,z \Box [P(x,y,z) \rightarrow \Box \neg P(z,y,x)]$	$x \rightarrow z$
c.	WEAK- $\pi_6$ -compatible:	$\forall x,y,z \Box [P(x,y,z) \rightarrow \Diamond P(z,y,x) \wedge \Diamond \neg P(z,y,x)]$	$y \rightarrow y$
d.	STRONG- $\pi_6$ -compatible:	$\forall x,y,z \Box [P(x,y,z) \rightarrow \Box P(z,y,x)]$	$z \rightarrow x$

The following examples illustrate the most common permutation patterns of ditransitive predicates attested in natural languages.

*Kam language* (Kadai family: Guizhou Province, P.R. of China)

- (33) mao<sup>33</sup> so<sup>323</sup> woŋ<sup>453</sup> nɛm<sup>31</sup> taŋ<sup>11</sup> ja<sup>53</sup> ʎa<sup>33</sup>. |  $P$  is AMBI- $\pi_1$   
 3P SG dry bucket water CL field DEM:DIST |  $P$  is NON- $\pi_1, -\pi_2, -\pi_3, -\pi_4, -\pi_5, -\pi_6$   
 ‘He dried the field of water with a bucket (i.e. water was removed from the field to dry it).’

- (34) nɛi<sup>31</sup> jən<sup>31</sup> lak<sup>31</sup> t<sup>h</sup>əu<sup>453</sup> tɕu<sup>11</sup> nɛa<sup>55</sup>. |  $P$  is AMBI- $\pi_1, -\pi_3$   
 mother lead, bring son PREP CL river |  $P$  is NON- $\pi_2, -\pi_4, -\pi_5, -\pi_6$   
 $P$  is WEAK- $\pi_1, -\pi_3$   
 ‘The mother brought her son to a river.’

- (35) mao<sup>33</sup> p<sup>h</sup>ja<sup>35</sup> k<sup>w</sup>aŋ<sup>323</sup> əu<sup>31</sup> nɛa<sup>11</sup>. |  $P$  is AMBI- $\pi_1, -\pi_6$   
 3P SG feed bowl rice 2P SG |  $P$  is NON- $\pi_2, -\pi_3, -\pi_4, -\pi_5$   
 $P$  is WEAK- $\pi_1, -\pi_6$   
 ‘He feeds you with a bowl of rice.’

*Lolo language* (Tibeto-Burman family: Yunnan Province, P.R. of China)

- (36) ɔ<sup>55</sup>no<sup>33</sup>sə<sup>33</sup> t<sup>h</sup>je<sup>21</sup> ɔ<sup>21</sup>mo<sup>33</sup> bɔ<sup>33</sup>lu<sup>21</sup> t<sup>h</sup>je<sup>21</sup> |  $P$  is AMBI- $\pi_1, -\pi_2, -\pi_3, -\pi_4, -\pi_5, -\pi_6$   
 name of man O-marker mother name of man B-marker |  $P$  is WEAK- $\pi_1, -\pi_2, -\pi_3, -\pi_4, -\pi_5, -\pi_6$   
 dzi<sup>33</sup> gə<sup>21</sup>.  
 hand over PRED:give  
 ‘Mom handed Onose over to Bolu.’

*English*

- (37) Boston is **closer** to New York than to Los Angeles.<sup>6</sup> |  $P$  is AMBI- $\pi_1, -\pi_2, -\pi_3, -\pi_4, -\pi_5, -\pi_6$   
 $P$  is NON- $\pi_2, -\pi_4, -\pi_5$   
 $P$  is STRONG- $\pi_1, -\pi_3, -\pi_6$

In analogy to (21) and (25), we can model four permutation sets for each ditransitive predicate  $P$ ,  $S_{ambi}(P)$ ,  $S_{non}(P)$ ,  $S_{weak}(P)$  and  $S_{strong}(P)$ .

**(38) Definition (Permutation sets of ditransitive predicates):**

- a.  $S_{ambi}(P) = \{\pi: \{x,y,z\} \rightarrow \{x,y,z\} \mid \pi \text{ bijective, AMBI-}\pi(P)\};$   
 -----  
 b.  $S_{non}(P) = \{\pi: \{x,y,z\} \rightarrow \{x,y,z\} \mid \pi \text{ bijective, NON-}\pi(P)\};$   
 c.  $S_{weak}(P) = \{\pi: \{x,y,z\} \rightarrow \{x,y,z\} \mid \pi \text{ bijective, WEAK-}\pi(P)\};$   
 d.  $S_{strong}(P) = \{\pi: \{x,y,z\} \rightarrow \{x,y,z\} \mid \pi \text{ bijective, STRONG-}\pi(P)\}.$

In addition to examples (33)-(37), consider the permutations sets of the following predicates.

<sup>6</sup> The predicate  $P$ : **closer** is NON- $\pi_2, -\pi_4, -\pi_5$  and STRONG- $\pi_1, -\pi_3, -\pi_6$  if the geographic positions of the three arguments are like for ‘Boston’, ‘New York’ and ‘Los Angeles’ in the real world. These permutations properties, however, may not hold if the geographic positions are different. The predicate  $P$ : **closer** is an ambiguous lexeme that covers several geographic relations. The permutation properties can be stated clearly to the extent that these geographic relations are specified.

- (39) **Examples:**
- a. For  $P$ : so<sup>323</sup> ‘dry’ in the Kam language, we have
    - $S_{ambi}(P) = \{\pi_1\}$
    - $S_{non}(P) = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6\}$
    - $S_{weak}(P) = \emptyset$
    - $S_{strong}(P) = \emptyset$
  - b. For  $P$ : jən<sup>31</sup> ‘lead’ in the Kam language, we have
    - $S_{ambi}(P) = \{\pi_1, \pi_3\}$
    - $S_{non}(P) = \{\pi_2, \pi_4, \pi_5, \pi_6\}$
    - $S_{weak}(P) = \{\pi_1, \pi_3\}$
    - $S_{strong}(P) = \emptyset$ .
  - c. For  $P$ : p<sup>h</sup>ja<sup>35</sup> ‘feed’ in the Kam language, we have
    - $S_{ambi}(P) = \{\pi_1, \pi_6\}$
    - $S_{non}(P) = \{\pi_2, \pi_3, \pi_4, \pi_5\}$
    - $S_{weak}(P) = \{\pi_1, \pi_6\}$
    - $S_{strong}(P) = \emptyset$
  - d. For  $P$ : dzi<sup>33</sup> ‘hand over’ in the Lolo language, we have
    - $S_{ambi}(P) = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6\}$
    - $S_{non}(P) = \emptyset$
    - $S_{weak}(P) = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6\}$
    - $S_{strong}(P) = \emptyset$ .
  - e. For  $P$ : *closer* in English, we have
    - $S_{ambi}(P) = \{\pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6\}$
    - $S_{non}(P) = \{\pi_2, \pi_4, \pi_5\}$
    - $S_{weak}(P) = \emptyset$
    - $S_{strong}(P) = \{\pi_1, \pi_3, \pi_6\}$ .

## 5. Mathematical properties of predicate-induced permutations

In §5.1, I develop the logical meta-language in which permutations are formalized. In §5.2, I define the generic form of permutation properties for  $n$ -place predicates and in §5.3, I elaborate on the notion of permutation group of degree  $n$ .

### 5.1 The Language *MTPL* (Modal-Tense Predicate Logic)

The notion of argument permutation requires a temporal and a modal component. The inclusion of both parameters in one analysis is of course not a new idea but has been routinely applied to linguistics since Dowty (1979)’s work on the English progressive aspect. However, in Dowty’s original approach and in that of a few other scholars the exact relationship between time and possible worlds was left undefined. In this paper, I adopt the notion of  $T \times W$ -frames. In modal logic, there are two ways of interpreting formulas through times and possible worlds. One involves Kamp-frames and the other  $T \times W$ -frames (cf. Thomason 1984, Wölfl 1999). Both formalizations differ on the question of whether the ordering of times is world-dependent or not. For Kamp-frames the ordering of times is world-dependent and for  $T \times W$ -frames it is world-independent. Let *MTPL* be the language of modal predicate logic with two intensional operators  $\Box$  (necessity) and  $\Diamond$  (possibility) (see Hintikka 1969). A model of MPL is a triple  $M = \langle D, T \times W, F \rangle$  such that

- (40) Definition:**
- a.  $D$  is a set of individuals;
  - b.  $T \times W$  is a  $T \times W$ -frame, i.e. a structure  $(W, T, <, \approx)$  where  $W$  and  $T$  are disjoint non-empty sets of possible worlds and time points;
    - $(T, <)$  a linear, irreflexive and transitive order;
    - $T$  is *open* (in the sense of topology, i.e. it has no terminal point);
    - $\approx$  is a relation in  $T \times W \times W$  such that
      - for all  $t \in T$   $\approx_t$  is an equivalence relation;
      - for all  $t, t' \in T$  and  $w, w' \in W$ , if  $w \approx_t w'$  and  $t' < t$  then  $w \approx_{t'} w'$ ;

In traditional modal predicate logic, constants and variables are interpreted not as referring to plain individuals but to “individual concepts” (see, for example, Aloni 2005: 508). Individual concepts are maps from time-world pairs  $(t, w)$  in  $T \times W$ , called *scenarios*, to individuals in  $D$ .

- c.  $F$  is a functor which maps each non-logical constant to interpretations:
  - For each constant  $c$ ,  $F(c): T \times W \rightarrow D$  is an assignment function;
  - For each n-ary predicate symbol  $P$ ,  $F(P): D^n \rightarrow \wp(T \times W)$ ;

Propositions are interpreted in a model  $M$  with respect to scenarios  $(t, w)$ , and an assignment function  $g$ , mapping variables to individual concepts in  $D^{T \times W}$ .

- (41) Definition:**
- a.  $(t, w) \models_{M,g} P(x_1, \dots, x_n)$  iff  $(t, w) \in F(P)(g(x_1), \dots, g(x_n))$
  - b.  $(t, w) \models_{M,g} \neg \phi$  iff not  $(t, w) \models_{M,g} \phi$
  - c.  $(t, w) \models_{M,g} \phi \wedge \psi$  iff both  $(t, w) \models_{M,g} \phi$  and  $(t, w) \models_{M,g} \psi$ ;
  - d.  $(t, w) \models_{M,g} \phi \vee \psi$  iff either  $(t, w) \models_{M,g} \phi$  or  $(t, w) \models_{M,g} \psi$  or both;
  - e.  $(t, w) \models_{M,g} \phi \rightarrow \psi$  iff if  $(t, w) \models_{M,g} \phi$  then  $(t, w) \models_{M,g} \psi$ ;
  - f.  $(t, w) \models_{M,g} \exists x \phi$  iff there is  $d \in D^{T \times W}$  such that  $(t, w) \models_{M,g[x/d]} \phi$ ;
  - g.  $(t, w) \models_{M,g} \forall x \phi$  iff for all  $d \in D^{T \times W}$  it is the case that  $(t, w) \models_{M,g[x/d]} \phi$ .

The notion of  $T \times W$ -frame induces a canonical accessibility relation on  $T \times W$ .

**(42) Definition (Canonical Accessibility Relation):**

$(t, w) < (t', w')$  iff  $t < t'$  and  $w \approx_t w'$ .

Having specified the canonical accessibility relation of the model  $M$ , we are equipped to interpret the intensional formulas.

- (41) Definition:**
- h.  $(t, w) \models_{M,g} \Box \phi$  iff for all  $(t', w')$ : if  $(t, w) < (t', w')$  then  $(t', w') \models_{M,g} \phi$
  - i.  $(t, w) \models_{M,g} \Diamond \phi$  iff there is  $(t', w')$  such that  $(t, w) < (t', w')$  and  $(t', w') \models_{M,g} \phi$ .

For a given model  $M$  of *MTPL* and an assignment function  $g$ , we will use the following abbreviation:

- (43) Abbreviation:**  $(t, w) \models \phi$  for  $(t, w) \models_{M,g} \phi$ .

This interpretation with scenarios can be illustrated for the following monotransitive predicate in the Kam language. The kinship predicate NONGX is strong-repeatable and asymmetric (the string NONGX in the romanized Kam orthography is pronounced as  $noŋ^{31}$ ). The predicate NONGX covers the English kinship terms ‘younger brother’, ‘younger sister’ and ‘younger cousin’ (Geary et al. 2003: 93).<sup>7</sup>

<sup>7</sup> The Kam kinship system is reminiscent of the Eskimo and Hawaiian naming systems in anthropology. In the same generation of EGO, there is no distinction made between the sex of kins and between parallel and cross cousins. The only lexicalized feature is the relative age of the kin in relation to EGO.

The predicate NONGX is strong-repeatable. Suppose that Bill is John's NONGX in scenario  $(t, w)$ , then in any scenario  $(t', w')$  accessible from  $(t, w)$  Bill is still John's NONGX. The predicate is asymmetric for the following reason. If in one scenario  $(t, w)$  Bill is the NONGX of John, then there is no scenario  $(t', w')$  which grows out of  $(t, w)$  and in which John is the NONGX of Bill.

## 5.2 Argument-permutation properties and permutation sets of degree $n$

The form of permutation properties for intransitive, monotransitive and ditransitive predicates is transparent from §4 and can be generalized into the following data. Let the following entities be given

- a. for a natural number  $n$ ,
  - an  $n$ -place predicate  $P$
  - $n$  arguments  $x_1, \dots, x_n$
- b. a bijective function  $\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$  (also called *permutation*).

We define four argument-permutation properties: AMBI- $\pi(P)$ , NON- $\pi(P)$ , WEAK- $\pi(P)$ , STRONG- $\pi(P)$ . The property AMBI- $\pi(P)$  expresses that arguments can undergo  $\pi$ -permutation in two unrelated (= independent) events, while NON- $\pi(P)$ , WEAK- $\pi(P)$  and STRONG- $\pi(P)$  state the necessity or possibility of  $\pi$ -permuted arguments in two consecutive (= dependent) events. The property AMBI- $\pi(P)$  measures the ambiguity with which semantic roles are syntactically encoded by the predicate  $P$ .

### (44) Definition (Generic form of argument-permutation properties):

- a. AMBI- $\pi$ -compatible:  $\forall x_1 \dots \forall x_n [\Diamond P(x_1, \dots, x_n) \wedge \Diamond P(x_{\pi(1)}, \dots, x_{\pi(n)})]$
- b. NON- $\pi$ -compatible:  $\forall x_1 \dots \forall x_n \Box [P(x_1, \dots, x_n) \rightarrow \Box \neg P(x_{\pi(1)}, \dots, x_{\pi(n)})]$
- c. WEAK- $\pi$ -compatible:  $\forall x_1 \dots \forall x_n \Box [P(x_1, \dots, x_n) \rightarrow \Diamond P(x_{\pi(1)}, \dots, x_{\pi(n)}) \wedge \Diamond \neg P(x_{\pi(1)}, \dots, x_{\pi(n)})]$
- d. STRONG- $\pi$ -compatible:  $\forall x_1 \dots \forall x_n \Box [P(x_1, \dots, x_n) \rightarrow \Box P(x_{\pi(1)}, \dots, x_{\pi(n)})]$

These second order predicates satisfy several properties. In (45a), they satisfy a dependency entailment: if a predicate can  $\pi$ -permute its arguments in two dependent events, then it can  $\pi$ -permute them also in two independent events. Propositions (45b+c) state that a predicate either cannot  $\pi$ -permute its arguments, weakly permutes its arguments or strongly permutes its arguments. Let  $P$ ,  $x_1, \dots, x_n$  and bijection  $\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\}$  be arbitrary.

- (45) **Theorem:**
- a. *Dependency entailment:* We have  $[\text{WEAK-}\pi(P) \text{ or } \text{STRONG-}\pi(P)] \Rightarrow \text{AMBI-}\pi(P)$ .
  - b. *Cumulatively exhaustive:* We have NON- $\pi(P)$  or WEAK- $\pi(P)$  or STRONG- $\pi(P)$ .
  - c. *Mutually exclusive:* We have
    - NON- $\pi(P) \Rightarrow$  not WEAK- $\pi(P)$  and not STRONG- $\pi(P)$ ,
    - WEAK- $\pi(P) \Rightarrow$  not NON- $\pi(P)$  and not STRONG- $\pi(P)$ ,
    - STRONG- $\pi(P) \Rightarrow$  not NON- $\pi(P)$  and not WEAK- $\pi(P)$ .

**Proof:** See appendix.

Any of the following sets is called *permutation sets of degree  $n$* .

### (46) Definition:

- a.  $S_{ambi}(P) = \{\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\} \mid \pi \text{ bijective, AMBI-}\pi(P)\}$ ;
- b.  $S_{non}(P) = \{\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\} \mid \pi \text{ bijective, NON-}\pi(P)\}$ ;
- c.  $S_{weak}(P) = \{\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\} \mid \pi \text{ bijective, WEAK-}\pi(P)\}$ ;
- d.  $S_{strong}(P) = \{\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\} \mid \pi \text{ bijective, STRONG-}\pi(P)\}$ .

The set  $\{\pi: \{1, \dots, n\} \rightarrow \{1, \dots, n\} \mid \pi \text{ bijective}\}$  of all permutations of degree  $n$  is denoted by  $S_n$  in the mathematical literature and has the cardinality  $n!$  (see Merris 2003: 141). (45b+c) guarantee that  $S_{non}(P)$ ,  $S_{weak}(P)$  and  $S_{strong}(P)$  provide a partition of  $S_n$ , i.e. that their mutually disjoint union is  $S_n$ .

### 5.3 Predicate-induced permutation groups of degree $n$

In mathematical combinatorics,  $S_n$  can be viewed as exhibiting an algebraic group structure supplied by function composition.

**(47) Definition:** A *group*  $(G, \circ)$  is a non-empty set  $G$  together with a function  $\circ: G \times G \rightarrow G$  satisfying the following laws:

- Law of associativity:  $\forall f, g, h \in G, f \circ (g \circ h) = (f \circ g) \circ h$ ,
- Law of neutral element:  $\exists e \in G, \forall f \in G, e \circ f = f \circ e = f$ ,
- Law of inverse element:  $\forall f \in G, \exists f^{-1} \in G, f \circ f^{-1} = f^{-1} \circ f = e$ .

**(48) Example:** For  $S_n$  we can define  $\circ: S_n \times S_n \rightarrow S_n$

$$\begin{aligned} (\pi, \mu) &\rightarrow \pi \circ \mu: \{1, \dots, n\} \rightarrow \{1, \dots, n\} \\ k &\rightarrow \pi(\mu(k)) \end{aligned}$$

$(S_n, \circ)$  is a group with  $\varepsilon_n$ , the *identity permutation*, as the neutral element.

Let us define the concept of a subgroup.

**(49) Definition:** Let  $(G, \circ)$  be a group and  $U \subseteq G$  a non-empty subset. We say that  $(U, \circ)$  is a subgroup of  $(G, \circ)$  iff

- $U$  is closed under  $\circ$  (i.e.  $\forall f, g \in U, f \circ g \in U$ ) inducing  $\circ|_U: U \times U \rightarrow U$ ,
- $(U, \circ|_U)$  is a group.

From this point on, our primary interest will be to investigate conditions under which  $S_{ambi}(P)$ ,  $S_{non}(P)$ ,  $S_{weak}(P)$  and  $S_{strong}(P)$  are subgroups of  $S_n$ . The subgroups of  $S_n$  are called *permutation group of degree  $n$* . Let us demonstrate the following lemma.

**(50) Lemma:** If  $G \subseteq S_n$  is a non-empty subset that is closed under function composition, then  $(G, \circ)$  is a subgroup of  $(S_n, \circ)$ .

**Proof:** See appendix.

**(51) Example:**  $S_3$  consists of 6 elements (§3.3). There are  $2^6 = 64$  subsets of  $S_3$  and 63 non-empty subsets. However, only six of them are closed. Besides  $S_3$ , there are only five proper subgroups of  $S_3$ :  $\{\varepsilon_3\}$ ,  $\{\varepsilon_3, (2,1,3)\}$ ,  $\{\varepsilon_3, (3,2,1)\}$ ,  $\{\varepsilon_3, (1,3,2)\}$ ,  $\{\varepsilon_3, (2,3,1), (3,1,2)\}$ . If we involve the format used in §3.3, we can also give the five proper subgroups as  $\{\pi_1\}$ ,  $\{\pi_1, \pi_3\}$ ,  $\{\pi_1, \pi_6\}$ ,  $\{\pi_1, \pi_2\}$ ,  $\{\pi_1, \pi_4, \pi_5\}$ .

We can show that  $S_{ambi}(P)$  is always a permutation group of degree  $n$ .

**(52) Lemma:**  $S_{ambi}(P) \subseteq S_n$  is always a subgroup of  $(S_n, \circ)$ .

**Proof:** See appendix.

Generally, it is not the case that  $S_{non}(P)$ ,  $S_{weak}(P)$  and  $S_{strong}(P)$  are subgroups of  $S_n$ . The next result we can establish relates to  $S_{strong}(P)$ .



**(53) Lemma:**  $S_{strong}(P) \subseteq S_n$  is either empty or a subgroup of  $(S_n, \circ)$ .

**Proof:** See appendix.

Now it is obvious that if  $S_{strong}(P) \subseteq S_n$  is a non-empty subgroup,  $S_{weak}(P)$  and  $S_{non}(P)$  cannot be subgroups of  $S_n$  because the neutral element  $\varepsilon_n$  would belong to  $S_{strong}(P)$  and thus could not be element of  $S_{weak}(P)$  and  $S_{non}(P)$ . For  $S_{weak}(P)$  there seems to be a similar result as lemma (53).

**(54) Lemma:**  $S_{weak}(P) \subseteq S_n$  is either empty or a subgroup of  $(S_n, \circ)$ .

**Proof:** See appendix.

From lemma (53) and lemma (54), we can deduce the following theorem.

**(55) Theorem of Predicate-Induced Permutation Groups of Degree n:**

Let  $n \geq 1$  and  $P$  be an  $n$ -place predicate. The following three mutually exclusive cases hold:

- a.  $S_{strong}(P) \subseteq S_n$  is a non-empty subgroup of  $(S_n, \circ)$  and  $S_{weak}(P) = \emptyset$ ;
- b.  $S_{strong}(P) = \emptyset$  and  $S_{weak}(P) \subseteq S_n$  is a non-empty subgroup of  $(S_n, \circ)$ ;
- c.  $S_{strong}(P) = \emptyset$ ,  $S_{weak}(P) = \emptyset$  and  $S_{non}(P) = S_n$ .

This pattern is exactly reflected in the examples of specific predicates that we illustrated in §4, see (22), (26), (39). Note that for (55a) and (55b), we do not state anything about  $S_{non}(P)$  which can be empty or non-empty. The important detail to keep in mind is that in both cases  $S_{non}(P)$  cannot form a subgroup of  $(S_n, \circ)$ .  $S_{non}(P)$  becomes a subgroup only in case (55c).

## 6. Permutation groups inform grammaticality judgments

The formal insights of §5 permit to represent natural language properties in terms of permutation groups. We revisit the language phenomena of §2 in order.

### 6.1 Quantificational aspect constrained by $S_{weak}(P) \neq \emptyset$

According to §2.1, the experiential and habitual aspect particles can be appended to a sentence in the Kam language if and only if the predicate of the sentence is weak-repeatable. As the idea of repeatability is associated with the identity permutation in two dependent (= consecutive) events, we can formulate the rule for quantificational aspect as follows.

(56) Grammaticality constraint on quantificational aspect:

An  $n$ -place predicate  $P$  is grammatically compatible with an experiential/habitual aspect operator if and only if  $S_{weak}(P)$  is a non-empty permutation subgroup of  $S_n$ .<sup>8</sup>

### 6.2 Inverse marking constrained by $S_{ambi}(P) = S_2$

In Kutenai and other Algonquian languages, inverse marking is a reversal system of subject/object marking only available if the sentence predicate allows arguments to undergo the identity and symmetry permutations in two independent events. This constraint is thus associated with the permutation group  $S_{ambi}(P)$ .

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<sup>8</sup> Because of theorem (55), this condition is equivalent to the requirement  $\varepsilon_n \in S_{weak}(P)$ .

(57) Grammaticality constraint on inverse marking:

A monotransitive predicate  $P$  is grammatically compatible with inverse marking if and only if  $S_{ambi}(P) = S_2$ .

### 6.3 Reciprocal constructions constrained by $S_{weak}(P) / S_{strong}(P) = S_2$

Virtually all languages have reciprocal constructions built on a reciprocal anaphor or a reciprocal verb affix. Reciprocal constructions always involve two dependent events or relations, one in which two arguments occur in a certain order, the other in which they are arranged in symmetric order. Reciprocal constructions are thus associated with  $S_{weak}(P)$  or  $S_{strong}(P)$ .

(58) Grammaticality constraint on reciprocal constructions:

A monotransitive predicate  $P$  is grammatically compatible with reciprocal constructions if and only if  $S_{weak}(P) = S_2$  or  $S_{strong}(P) = S_2$ .

It is not adequate to characterize reciprocal constructions in terms of the permutation group  $S_{ambi}(P)$ . For the predicate  $P$ : *bury*, for example, we have  $S_{ambi}(P) = S_2$ , but  $P$  cannot be fed into a reciprocal construction (i.e. *bury each other*).

### 6.4 Free word order languages constrained by $|S_{ambi}(P)|$

In free word order languages, ambiguity between (human) NP arguments is resolved through case marking. Lolo (§2.3 and §2.4) is special in that it involves case marking ( $thie^{21}$ ) only if there is ambiguity between arguments or, put differently, if the predicate allows permutations of its arguments in independent events. Case marking ( $thie^{21}$ ) in Lolo hence depends on  $S_{ambi}(P)$  and its cardinality. According to §2.3 and §2.4, four cases must be distinguished.

(59) Grammaticality constraint on simple clauses:

A predicate  $P$  in Lolo conditions case-marking in the following way:

- a. If  $|S_{ambi}(P)| = 1$ , then  $thie^{21}$  need not be involved; if involved, it functions as focus marker.
- b. If  $|S_{ambi}(P)| = 2$ , then  $thie^{21}$  is used as case marker once after the NP identified as the direct object (O); it does not function as focus marker.
- c. If  $|S_{ambi}(P)| = 6$ , then  $thie^{21}$  is used as case marker twice after two noun phrases; the direct object (O) is the first NP marked by  $thie^{21}$  and the beneficiary (B) the second.
- d. If  $|S_{ambi}(P)| = 12$ , then  $thie^{21}$  is used after the second, third and fourth NP; the first NP not marked by  $thie^{21}$  is the CAUSER, the second NP is the CAUSEE, the third NP is the direct object (O) and the fourth NP the beneficiary (B).

## 7. Conclusion

This paper develops mathematical tools for combinatorial phenomena of natural languages. Each sentence predicate correlates with two finite permutation groups which measure the predicate's ability of permuting its NP arguments in two independent events (first group) and in two dependent events (second group). These permutation groups conceptualize complex grammaticality phenomena that previously were not identified to be combinatorial in nature.

At a methodological level, the paper started by cataloguing several linguistic combinatorial phenomena (quantificational aspect, inverse marking, free word order languages). It proposes to view these phenomena as mathematical combinatorial problems for which it develops a mathematical representation theorem (see 55). This theorem is reinterpreted in natural language data in a way that enlightens the description of grammaticality properties.

The work presented in this paper naturally extends in a number of ways. Linguistically, we may

look for syntactic devices that push the dimension of the permutation group into a higher range. (In this paper, the highest dimension of permutation groups backed up by natural language data is four.) Serial verb constructions are syntactic devices that exist in many language families of the world except for Indo-European (Aikhenvald 2006). Serial verb constructions consist of two or more predicates whose semantic relation is not morphologically marked with conjunctions. Some of these constructions are “typed” in the sense that one predicate, as a result of grammaticalization, triggers the presence of a second predicate. These predicate pairs can be viewed as single complex predicates that take up to four or five arguments. The kind of permutation group associated with this complex predicate could capture important linguistic properties of serial verb constructions (e.g. argument sharing).

Furthermore, other combinatorial notions could be launched following the ideas developed in this paper. I mentioned equivalence relations at the beginning of §1 which are defined as reflexive, symmetric and transitive relations. The traditional definition of a transitive relation  $R$  is:  $R(X, Y)$  and  $R(Y, Z)$  implies  $R(X, Z)$ . In natural languages, there are at least three types of transitive relations which can be modeled by the concepts of NON, WEAK and STRONG proposed in this paper.

(60) *Father* is NON-transitive

- a. John is Bill’s father.
- b. Bill is Peter’s father.
- c. John is Peter’s father.

(61) *Beat* is WEAK-transitive

- a. John beats Bill.
- b. Bill beats Peter.
- c. John beats Peter.

(62) *Older* is STRONG-transitive

- a. John is older than Bill.
- b. Bill is older than Peter.
- c. John is older than Peter.

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#### **Appendix**

In this section, I provide mathematical proofs of results contained in the main text. To facilitate the reading of the paper, the proofs were not included there.

- (45) Theorem:**
- a. *Dependency entailment:* We have  $[\text{WEAK-}\pi(P) \text{ or } \text{STRONG-}\pi(P)] \Rightarrow \text{AMBI-}\pi(P)$ .
  - b. *Cumulatively exhaustive:* We have  $\text{NON-}\pi(P) \text{ or } \text{WEAK-}\pi(P) \text{ or } \text{STRONG-}\pi(P)$ .
  - c. *Mutually exclusive:* We have
    - $\text{NON-}\pi(P) \Rightarrow \text{not WEAK-}\pi(P) \text{ and not STRONG-}\pi(P)$ ,
    - $\text{WEAK-}\pi(P) \Rightarrow \text{not NON-}\pi(P) \text{ and not STRONG-}\pi(P)$ ,
    - $\text{STRONG-}\pi(P) \Rightarrow \text{not NON-}\pi(P) \text{ and not WEAK-}\pi(P)$ .

**Proof:**

- a. *Dependency inclusion:*
  - Suppose  $\text{WEAK-}\pi(P)$ . Natural language predicates have the basic function of predicating over NP arguments. For all  $x_1 \dots x_n$  there is thus  $(t, w)$  such that  $(t, w) \models P(x_1, \dots, x_n)$ . From the definition of  $\text{WEAK-}\pi(P)$ , it follows that there is a scenario  $(t', w')$  with  $(t, w) < (t', w')$  such that  $(t', w') \models P(x_{\pi(1)}, \dots, x_{\pi(n)})$ . From this, we may conclude that  $\text{AMBI-}\pi(P)$ .
  - Suppose  $\text{STRONG-}\pi(P)$ . Again, for all  $x_1 \dots x_n$  there is a scenario  $(t, w)$  such that  $(t, w) \models P(x_1, \dots, x_n)$ . All we need to continue in our argumentation is to assume that there is another  $(t', w')$  accessible through  $(t, w)$ . This is warranted because  $T$  is topologically open as assumed in (38b). From the definition of  $\text{STRONG-}\pi(P)$ , it follows that for  $(t', w')$  we have  $(t', w') \models P(x_{\pi(1)}, \dots, x_{\pi(n)})$ . We may thus conclude that  $\text{AMBI-}\pi(P)$ .
- b. *Cumulatively exhaustive:* Suppose that  $(t, w) \models P(x_1, \dots, x_n)$  for an arbitrary scenario  $(t, w)$ .  $P$  either satisfies  $\text{STRONG-}\pi(P)$  or does not satisfy it. Suppose that it does not satisfy it. It follows that there is  $(t', w')$  such that  $(t, w) < (t', w')$  and  $(t', w') \models \neg P(x_{\pi(1)}, \dots, x_{\pi(n)})$ . There are then two cases that need to be distinguished: either for all  $(t'', w'')$  with  $(t, w) < (t'', w'')$  we have  $(t'', w'') \models \neg P(x_{\pi(1)}, \dots, x_{\pi(n)})$ , in which case it follows that  $\text{NON-}\pi(P)$ , or there is another  $(t'', w'')$  with  $(t, w) < (t'', w'')$  and  $(t'', w'') \models P(x_{\pi(1)}, \dots, x_{\pi(n)})$ , in which case we have  $\text{WEAK-}\pi(P)$ . In case that the premise does not hold we have  $\text{STRONG-}\pi(P)$ .
- c. *Mutually exclusive:* Suppose that  $(t, w) \models P(x_1, \dots, x_n)$  for an arbitrary scenario  $(t, w)$ .
  - If  $\text{NON-}\pi(P)$ , then for all  $(t', w')$  with  $(t, w) < (t', w')$  we have  $(t', w') \models \neg P(x_{\pi(1)}, \dots, x_{\pi(n)})$ . From the definition of the weak and strong properties, it ensues that not  $\text{WEAK-}\pi(P)$  and not  $\text{STRONG-}\pi(P)$ .
  - If  $\text{WEAK-}\pi(P)$ , there are two scenarios  $(t', w')$  and  $(t'', w'')$  with  $(t, w) < (t', w')$  and  $(t, w) < (t'', w'')$  such that  $(t', w') \models P(x_{\pi(1)}, \dots, x_{\pi(n)})$  and  $(t'', w'') \models \neg P(x_{\pi(1)}, \dots, x_{\pi(n)})$ . It immediately follows that not  $\text{NON-}\pi(P)$  and not  $\text{STRONG-}\pi(P)$ .
  - If  $\text{STRONG-}\pi(P)$ , then for all  $(t', w')$  with  $(t, w) < (t', w')$  we have  $(t', w') \models P(x_{\pi(1)}, \dots, x_{\pi(n)})$ . It immediately follows that not  $\text{NON-}\pi(P)$  and not  $\text{WEAK-}\pi(P)$ .

**(50) Lemma:** If  $G \subseteq S_n$  is a non-empty subset that is closed under function composition, then  $(G, \circ)$  is a subgroup of  $(S_n, \circ)$ .

**Proof:**

Let  $\pi \in G$ . We can define recursively the permutation  $\pi^m$  ( $\pi$  power  $m$ ). We may pose  $\pi^0 = \varepsilon_n$  and  $\pi^m = \pi \circ \pi^{m-1}$  for  $m \geq 1$ . In mathematical combinatorics,  $o(\pi)$ , called the *order* of  $\pi$ , is defined as the smallest positive integer  $k$  such that  $\pi^k = \varepsilon_n$  which can be proven to exist (see Merris 2003: 186-187). As  $G$  is closed under  $\circ$ ,  $\varepsilon_n = \pi^k \in G$  serving as the neutral element of  $G$ . Furthermore, it is obvious that  $\pi^{-1} = \pi^{k-1}$  is the inverse element of  $\pi$  in  $G$  (and also in  $S_n$ ). Furthermore it is obvious that the law of associativity also holds for  $(G, \circ)$ .

**(52) Lemma:**  $S_{\text{ambi}}(P) \subseteq S_n$  is always a subgroup of  $(S_n, \circ)$ .

**Proof:**

All we need to show is that  $S_{\text{ambi}}(P)$  is closed under  $\circ$ . To show this point, let  $\pi, \mu \in S_{\text{ambi}}(P)$ . We thus have  $\forall x_1 \dots \forall x_n [\Diamond P(x_1, \dots, x_n) \wedge \Diamond P(x_{\pi(1)}, \dots, x_{\pi(n)})]$  and  $\forall y_1 \dots \forall y_n [\Diamond P(y_1, \dots, y_n) \wedge \Diamond P(y_{\mu(1)}, \dots, y_{\mu(n)})]$ . Posing  $y_1 = x_{\pi(1)}, \dots, y_n = x_{\pi(n)}$ , we may obtain

the simplified  $\forall x_1 \dots \forall x_n [\Diamond P(x_1, \dots, x_n) \wedge \Diamond P(x_{\pi(\mu(1))}, \dots, x_{\pi(\mu(n))})]$ . We have thus shown that  $\text{AMBI-}\pi^\circ\mu(P)$  or that  $\pi^\circ\mu \in S_{\text{ambi}}(P)$ . In other words,  $S_{\text{ambi}}(P)$  is closed.

**(53) Lemma:**  $S_{\text{strong}}(P) \subseteq S_n$  is either empty or a subgroup of  $(S_n, \circ)$ .

**Proof:** We must demonstrate that if  $S_{\text{strong}}(P)$  is non-empty, then it is closed under  $\circ$ . To show this point, let  $\pi, \mu \in S_{\text{strong}}(P)$ , let  $x_1, \dots, x_n$  be  $n$  arguments and  $(t, w)$  a scenario such that  $(t, w) \models P(x_1, \dots, x_n)$ . As  $\text{STRONG-}\mu(P)$ , for all  $(t', w')$  with  $(t, w) < (t', w')$  we have  $(t', w') \models P(x_{\mu(1)}, \dots, x_{\mu(n)})$ . As  $\text{STRONG-}\pi(P)$ , for all  $(t'', w'')$  with  $(t', w') < (t'', w'')$  we have  $(t'', w'') \models P(x_{\pi(\mu(1))}, \dots, x_{\pi(\mu(n))})$ . As the accessibility relation  $<$  is transitive, it is true that for all  $(t'', w'')$  with  $(t, w) < (t'', w'')$  we have  $(t'', w'') \models P(x_{\pi(\mu(1))}, \dots, x_{\pi(\mu(n))})$ . Therefore we have proven that  $\text{STRONG-}\pi^\circ\mu(P)$  and that  $S_{\text{strong}}(P)$  is closed.

**(54) Lemma:**  $S_{\text{weak}}(P) \subseteq S_n$  is either empty or a subgroup of  $(S_n, \circ)$ .

**Proof:** Again, all we need to establish is that if  $S_{\text{weak}}(P)$  is non-empty, then it is closed under  $\circ$ . Let therefore  $\pi, \mu \in S_{\text{weak}}(P)$ , let  $x_1, \dots, x_n$  be  $n$  arguments and  $(t, w)$  a scenario such that  $(t, w) \models P(x_1, \dots, x_n)$ . As  $\text{WEAK-}\mu(P)$ , there are two scenarios  $(t_1, w_1)$  and  $(t_2, w_2)$  with  $(t, w) < (t_1, w_1)$  and  $(t, w) < (t_2, w_2)$  such that  $(t_1, w_1) \models P(x_{\mu(1)}, \dots, x_{\mu(n)})$  and  $(t_2, w_2) \models \neg P(x_{\mu(1)}, \dots, x_{\mu(n)})$ . For  $(t_1, w_1)$  there are two other scenarios  $(t_3, w_3)$  and  $(t_4, w_4)$  accessible from  $(t_1, w_1)$  such that  $(t_3, w_3) \models P(x_{\pi(\mu(1))}, \dots, x_{\pi(\mu(n))})$  and  $(t_4, w_4) \models \neg P(x_{\pi(\mu(1))}, \dots, x_{\pi(\mu(n))})$ . As the accessibility relation  $<$  is transitive,  $(t_3, w_3)$  and  $(t_4, w_4)$  are also accessible from  $(t, w)$ . We have thus shown the existence of two scenarios, one that satisfies  $P(x_{\pi(\mu(1))}, \dots, x_{\pi(\mu(n))})$ , the other that rejects it. It follows that  $\text{WEAK-}\pi^\circ\mu(P)$  and that  $S_{\text{weak}}(P)$  is closed.

#### List of abbreviations

1P SG	First person singular pronoun
2P SG	Second person singular pronoun
3P SG	Third person singular pronoun
3P PL	Third person plural pronoun
B	Beneficiary (or recipient)
CL	Classifier
DEM:DIST	Demonstrative: distal distance to Speaker
DP	Dynamic perfect
EXP	Experiential marker
HAB	Habitual marker
IDE	Ideophone
NUM:3	Number and its value
INDIC	Indicative
INV	Inverse affix
O	Object
OBV	Obviative case marking
PREP	Preposition
RECL	Reciprocal anaphor
S	Subject (or agent)
V	Verb

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