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The Locally Free Relatively Filtered Diagram as an Inductive Completion of a System of Choice*

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Abstract. Guitart and Lair [5] have established the existence of Locally Free Diagrams, which can be seen as a purely categorical version of the solution set condition, and of the Lowenheim–Skolem theorem. Their proof is based on a transfinite construction by saturation. An iterative principle is established, but the construction is not effective for every step. The thesis of Gerner [3] contains a more effective proof for the existence of Locally Free Diagrams (with the restriction that the projective bases of the sketch S must all be finite). But the problem of [3] lies in the impossibility to name concretely the elements of the Locally Free Diagrams. The present paper will provide a new construction of the Locally Free Diagram in which the effective and the non-effective part will be much more separated (again the projective bases must all be finite). This construction represents a notable improvement with regard to the proof of [3] allowing the concrete designation of the elements of the Locally Free Diagrams. Furthermore we show that the construction is relatively filtered (i.e. satisfies the "filtered"-property).

Key words: sketches, completions, free structures.

1. Prerequisites

1.1. Motivation

1.1.1. *Free Structures*

In Algebra, free structures have been studied with interest for some time: the free monoid generated by an alphabet, the abelian group generated by a set, etc. In all these cases the situation is the same: there is a set X on which we want to construct an algebraic structure of a given type such that for any function from X to an algebraic structure M of this type there is a unique factorization property for the free algebraic structure $F(X)$ generated by X. We can express this with the following formula:

$$Hom(X, M) \cong Hom(F(X), M).$$

In the case of the monoids we can effectively construct the elements of the free monoid generated by an alphabet as the words on this alphabet.

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